PERFORMANCE EVALUATION OF HIGH RATE SPACE TIME TRELLIS CODED MODULATION USING GAUSS-CHEBYSHEV QUADRATURE TECHNIQUE

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***Abstract*— High rate space time trellis coded modulation (HR-STTCM) is an example of space time codes that combines the idea used in trellis coded modulation (TCM) design i.e. signal set expansion and set partitioning into its construction. HR-STTCM construction is based on the concatenation of an inner TCM encoder and outer space time block code. Here we use a simple numerical analysis method i.e. Gauss Chebyshev quadrature technique to obtain a closed form expression for the pairwise error probability of the code.**

***Index Terms*— Pairwise error probability, super orthogonal codes, space time codes, set partitioning, bit error rate, fading channel, trellis coded modulation.**

I. INTRODUCTION

In [1] High rate space time trellis coded modulation (HR-STTCM) was introduce as a space time coding scheme that has higher coding advantage when compared to the earlier design of space time trellis coded modulation [2], [3] and [4]. The advantage of the construction in [1] is that the standard technique for designing good trellis coded modulation codes [7], such as the classic set partitioning concept, can be adopted to realize the high rate space time trellis coded modulation design with large coding gain.

A parameterized class of space time code was introduce in [5], i.e. supper orthogonal space time trellis code (SOSTTC), which give a systematic approach in the design of high rate space time trellis coded modulation.

The SOSTTC does not only provide a scheme that has an improvement in the coding gain when compared with the original space time trellis coded modulations, but it answers the question of a systematic design for any rate, number of states and the maximization of coding gain. This matrix expansion in HR-STTCM corresponds to the angle multiplication in the SOSTTC. This means that the identity matrix multiplication with the original Alamouti code [6] corresponds to the angular multiplication in the SOSTTC.

For example, the multiplication of $diag[1,-1]$ with Alamouti code in HR-STTCM corresponds to $\theta = \pi$ in the orthogonal transmission matrix of the SOSTTC shown in (1) below.

$$A\left(x_1, x_2, \theta\right) = \begin{pmatrix} x_1 e^{j\theta} & x_2 \\ -x_2^* e^{j\theta} & x_1^* \end{pmatrix} \quad (1)$$

where $(*)$ stands for conjugate and $x_i \in e^{j\frac{2\pi a}{m}}$, $i = 1,2$.

When $\theta = 0$, (1) becomes the Alamouti code. For an m-PSK constellation with constellation signal represented by $e^{j\frac{2\pi a}{m}}$, $a = 0,1,\ldots,m-1$, one can pick $\theta = 2\pi a'/m$, where $a' = 0,1,\ldots,m-1$.

Our interest in this paper is in the pairwise error probability (PEP) and the average bit error probability (BEP) in both fast and slow (quasi-static) fading channel of the HR-STTCM using the orthogonal transmission matrix shown in (1).

In [4], performance criteria for space time codes were derived based on an upper bound on the PEP for both quasi-static and flat fading channels. Although the upper bound derived in [4] allows to considering all possible error events and gives a final expression for the average error probability terms, it is too loose for most signal-to noise ratio range.

Several research works [8, 9, 10] have been done to obtain a tighter bound for most space time codes using various methods and based on these expressions an analytical estimate for the bit error probability can be computed, taking into account dominant error event.

The closed form expression of the PEP for a space time trellis code was derived in [9] based on the residual method, which has been used previously in the performance analysis of trellis coded modulation. The derivation in [9] shows that the exact PEP is the upper bound derived in [4] modified by a correcting factor given by the second product term whose value depends on the poles of the characteristic function of the quadratic form of the complex Gaussian random variable.

In [10], the moment generating function previously used for the analysis of uncoded and coded digital communication over fading channels using only a single transmitter is applied to provide a closed form expression of the PEP for SOSTTC systems with multiple transmitters. The method used in [10]

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has an advantage of allowing for direct evaluation of the transfer function upper bound on the BEP. A different approach to the exact calculation of the PEP with less computational difficulty is presented here. This approach is based on the Gauss-Chebyshev quadrature technique.

The paper is organized as follows. In Section II, we discuss the general transmission model of the high rate space time trellis coded modulation and the channel model. Section III, the derivation of the PEP using the Gauss-Chebyshev quadrature technique is described and also numerical example using the derived closed form PEP expression for different error length is enumerated. In Section IV, we use the PEP obtained in Section III to estimate the average bit error probability for both quasi-static and fast fading channel. Section V concludes the paper with discussion on the results obtained from the numerical examples of both the PEP and the average bit error probability.

II. SYSTEM MODEL

We consider a transmission system of n_t transmit antennas and n_r receive antennas. The input binary data streams are first fed into an outer trellis code modulation encoder to generate a sequence of complex modulated symbols. The complex modulated symbols $x_i \ (i = 1, 2, ..., n_t)$ are then fed into an inner space time block encoder to generate the orthogonal transmitted code matrix (1). We define $x_{n_t}^{(n)}$ as the complex valued modulated symbol transmitted from the n_tth transmit antenna in the nth signaling interval and $h_{ij}^{(n)}$ is the channel coefficient from the ith transmit antenna to the jth receive antenna at the same signaling interval, $i \in \{1, 2, \ldots, n_t\}$, $j \in \{1, 2, \ldots, n_r\}$.

Assuming that the channel state information is known at the receiver, the corresponding set of successive signal sample at the receiver at two output time is given by:

$$\begin{aligned} r_l^{(n)} &= h_{l1}^{(n)} x_1^{(n)} e^{j\theta^{(n)}} + h_{l2}^{(n)} x_2^{(n)} + \eta_l^{(n)} \\ r_{l+n_r}^{(n)} &= h_{l1}^{(n)} \left(-x_2^{(n)}\right)^* e^{j\theta^{(n)}} + h_{l2}^{(n)} \left(x_1^{(n)}\right)^* + \eta_{l+n_r}^{(n)} \end{aligned} \tag{2}$$

where $l = 1, 2, ..., n_r$ and $\eta_i^{(n)}$ are independently identical distributed complex zero mean Gaussian noise samples, each sample with $\sigma^2/2$ per dimension. We assume that the channel elements undergo Rayleigh fading.

III PAIRWISE ERROR PROBABILITY

A. Derivations

To evaluate the PEP, i.e. the probability of choosing the codeword sequence $\tilde{X} = \left(\tilde{x}_1^{(1)}, \tilde{x}_2^{(1)}, ..., \tilde{x}_{n_t}^{(1)}\ \tilde{x}_1^{(2)}, \tilde{x}_2^{(2)}, ..., \tilde{x}_{n_t}^{(2)} .... \tilde{x}_1^{(N)}, \tilde{x}_2^{(N)}, ..., \tilde{x}_{n_t}^{(N)}\right)$ when in fact the codeword sequence, $X = \left(x_1^{(1)}, x_2^{(1)}, ..., x_{n_t}^{(1)}\ x_1^{(2)}, x_2^{(2)}, ..., x_{n_t}^{(2)} ... x_1^{(N)}, x_2^{(N)}, ..., x_{n_t}^{(N)}\right)$ was transmitted, we use the maximum likelihood metric corresponding to the correct path given by:

$$\begin{aligned} m(r, X) = \sum_{n=1}^{N} \sum_{l=1}^{n_r} \Bigg[&\left| r_l^{(n)} - \left(h_{l1}^{(n)} x_1^{(n)} e^{j\theta^{(n)}} + h_{l2}^{(n)} x_2^{(n)} \right) \right|^2 \\ &+ \left| r_{l+N_r}^{(n)} - \left(h_{l1}^{(n)} \left(-x_2^{(n)}\right)^* e^{j\theta^{(n)}} + h_{l2}^{(n)} \left(x_1^{(n)}\right)^* \right) \right|^2 \Bigg] \end{aligned} \tag{3}$$

The above is based on an observation of N blocks ($2N$ symbols), where each is described by (2). For the incorrect path, the corresponding metric is given by (3) with $x_i^{(n)}, i = 1, 2$ and $\theta^{(n)}$ replaced by $\tilde{x}_i^{(n)}, i = 1, 2$ and $\tilde{\theta}^{(n)}$ respectively.

The realization of the PEP over the entire frame length and for a given channel coefficient is given by:

$$\begin{aligned} P\left(X \to \tilde{X} \middle| H\right) &= \Pr\left\{ m(r, X) \ > \ m\left(r, \tilde{X}\right) \middle| H \right\} \\ &= \Pr\left\{ m(r, X) - m\left(r, \tilde{X}\right) > 0 \middle| H \right\} \end{aligned} \tag{4}$$

Substituting (3) and the corresponding expression for $m\left(r, \tilde{X}\right)$ into (4) and simplifying gives:

$$\begin{aligned} P\left(X \to \tilde{X} \middle| H\right) &= \Pr\left\{ \sum_{n=1}^{N} \sum_{l=1}^{n_r} \left[|A|^2 + |B|^2 \right] > 0 \middle| H \right\} \\ &= \Pr\left\{ \sum_{n=1}^{N} \sum_{l=1}^{n_r} \left\| H_l^{(n)} \Delta_n \right\|^2 > 0 \middle| H \right\} \end{aligned} \tag{5}$$

where

$$A = h_{l1}^{(n)} \left(\tilde{x}_1^{(n)} e^{j\tilde{\theta}^{(n)}} - x_1^{(n)} e^{j\theta^{(n)}} \right) + h_{l2}^{(n)} \left(\tilde{x}_2^{(n)} - x_2^{(n)} \right)$$

$$B = -h_{l1}^{(n)} \left(\tilde{x}_2^{(n)} e^{-j\tilde{\theta}^{(n)}} - x_2^{(n)} e^{-j\theta^{(n)}} \right)^* + h_{l2}^{(n)} \left(\tilde{x}_1^{(n)} - x_1^{(n)} \right)$$

and Δ_n is given as the codeword sequence matrix that characterize the HR-STTCM and its expression is given in (6) below and $H_l^{(n)} = \left[h_{l1}^{(n)} \quad h_{l2}^{(n)} \right]$.

$$\Delta_n = \begin{bmatrix} x_1^{(n)} e^{j\theta^{(n)}} - \tilde{x}_1^{(n)} e^{j\tilde{\theta}^{(n)}} & \left(-x_2^{(n)}\right)^* e^{j\theta^{(n)}} - \left(-\tilde{x}_2^{(n)}\right)^* e^{j\tilde{\theta}^{(n)}} \\ x_2^{(n)} - \tilde{x}_2^{(n)} & \left(x_1^{(n)}\right)^* - \left(\tilde{x}_1^{(n)}\right)^* \end{bmatrix} \tag{6}$$

The conditional PEP given in (5) can be expressed in terms of the complementary error function [11] as shown in (7).

$$P\left(X \to \tilde{X} \middle| H\right) = \frac{1}{2} erfc\left(\sqrt{\frac{E_s}{4N_0} \sum_{l=1}^{n_r} H_l \Delta \Delta^H H_l^H} \right) \tag{7}$$

The function $\Delta\Delta^H$ is a diagonal matrix of the form shown in (8). This represents the codeword sequence matrix for the entire frame length. $(g)^H$ represent the conjugate transpose of the matrix element and E_s/N_0 stands for the SNR per symbol.

$$\Delta\Delta^H = \begin{bmatrix} \Delta_1\Delta_1^H & 0 & \cdots & \cdots & 0 \\ 0 & \Delta_2\Delta_2^H & 0 & \vdots & 0 \\ 0 & 0 & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \Delta_{N-1}\Delta_{N-1}^H & 0 \\ 0 & 0 & \cdots & 0 & \Delta_N\Delta_N^H \end{bmatrix} \quad (8)$$

Complementary error function, as defined integrally in [12, 7.4.11] is given by:

$$erfc(b) = \frac{2}{\pi}\int_0^\infty \frac{e^{-b^2(t^2+1)}}{t^2+1}dt \quad (9)$$

Enumerating (7) using (9), we can then express the conditional PEP as an integral. Averaging the integral, one gets:

$$P(X \to \hat{X}) = \frac{1}{\pi}E\left[\int_0^\infty \frac{\exp\left[-(t^2+1)\frac{E_s}{4N_0}\sum_{l=1}^{n_r} H_l\Delta\Delta^H H_l^H\right]}{t^2+1}dt\right] \quad (10)$$

The above expression (10) can be simplified further using [8, eqn. (15)] to give

$$P(X \to \hat{X}) = \frac{1}{\pi}\int_0^\infty \frac{1}{t^2+1}\prod_{l=1}^{n_r}\frac{1}{\det[G]_l}dt \quad (11)$$

where $G = I_{n_t} + \frac{E_s}{4N_0}\Delta\Delta^H(t^2+1)$.

The above expression (11) can be approximated with a Gauss-Chebyshev quadrature formula, and details are given in the Appendix leading to the following:

$$P(X \to \hat{X}) = \frac{1}{2k}\sum_{i=1}^{k}\prod_{l=1}^{n_r}\frac{1}{\det\left[I_{n_T} + \frac{E_s}{4N_0}\Delta\Delta^H \sec^2\frac{(2i-1)\pi}{4k}\right]_l} + R_k \quad (12)$$

k is a small positive integer. As k increases the remainder term R_k becomes negligible.

For slow fading (quasi-static) case, the channel coefficient are assumed to be constant for the entire frame duration, but varies from frame to frame, (10) therefore result in:

$$P(X \to \hat{X}) = \frac{1}{2k}\sum_{i=1}^{k}\prod_{l=1}^{n_r}\frac{1}{\det[G_1]_l} + R_k \quad (13)$$

where $G_1 = I_{n_T} + \frac{E_s}{4N_0}\sum_{n=1}^{N}\Delta_n\Delta_n^H \sec^2\frac{(2i-1)\pi}{4k}$

For fast fading channel, the channel coefficient are kept constant for one codeword matrix and the change from matrix to matrix in a random manner, (10) will then result in:

$$P(X \to \hat{X}) = \frac{1}{2k}\sum_{i=1}^{k}\prod_{n=1}^{N}\prod_{l=1}^{n_r}\frac{1}{\det[G_2]_l} + R_k \quad (14)$$

where $G_2 = I_{n_T} + \frac{E_s}{4N_0}\Delta_n\Delta_n^H \sec^2\frac{(2i-1)\pi}{4k}$

B Numerical Examples

As an example, we consider the rate $r=1$ BPSK 2-state code [5, Fig. 1], whose trellis diagram is illustrated in Fig. 1, where two sets, each containing two pairs of BPSK symbols is assigned to each state, i.e. there is a pair of parallel path between each pair of states. The labeling $(s,l)/A(x_i, x_j, \theta)$ along each branch of the trellis refers to the pair of input BPSK symbols (s,l) and the corresponding output symbol function $A(x_i, x_j, \theta)$ using (1) to generate the orthogonal matrix.

First we consider the parallel path i.e. N = 1, evaluating (8) and substituting it into (13) and (14), we can observe that the PEP for this error event is the same for both fast and slow fading channel. The codeword matrix is given by:

$$\Delta_1 = \begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix}; \quad \Delta_1\Delta_1^H = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} \quad (15)$$

Also we consider an error event path of length N=2 with respect to the all zero path as the correct one. From the trellis diagram we have that
$x_1^{(1)} = x_2^{(1)} = x_1^{(2)} = x_2^{(2)} = +1$, $\tilde{x}_1^{(1)} = \tilde{x}_2^{(2)} = +1$, $\tilde{x}_2^{(1)} = \tilde{x}_1^{(2)} = -1$, $\theta^{(1)} = \tilde{\theta}^{(1)} = \theta^{(2)} = 0$ and $\tilde{\theta}^{(2)} = \pi$.
Evaluating the element of the matrix in (8) Δ_1 and Δ_2 give;

$$\Delta_1 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}; \quad \Delta_1\Delta_1^H = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$
$$\Delta_2 = \begin{bmatrix} 0 & -2 \\ 0 & 2 \end{bmatrix}; \quad \Delta_2\Delta_2^H = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \quad (16)$$

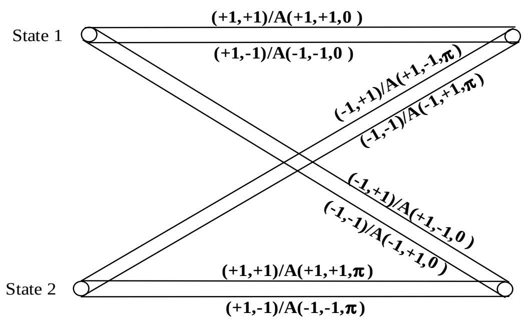

Fig. 1. Trellis diagram for rate 1, 2-state BPSK HR-STTCM

We also consider an error event of length 3 with respect to the all zero path as the correct one. From the trellis diagram we have:
$x_1^{(1)} = x_2^{(1)} = x_1^{(2)} = x_2^{(2)} = x_1^{(3)} = x_2^{(3)} = \tilde{x}_1^{(1)} = \tilde{x}_2^{(3)} = +1$, $\tilde{x}_2^{(1)} = \tilde{x}_1^{(2)} = \tilde{x}_2^{(2)} = \tilde{x}_1^{(3)} = -1$, $\theta^{(1)} = \tilde{\theta}^{(1)} = \theta^{(2)} = \theta^{(3)} = 0$ and $\tilde{\theta}^{(2)} = \tilde{\theta}^{(3)} = \pi$.
Evaluating the element of the matrix (8) Δ_1, Δ_2 and Δ_3 gives;

$$\Delta_1 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}; \quad \Delta_1 \Delta_1^H = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$
$$\Delta_2 = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}; \quad \Delta_2 \Delta_2^H = \begin{bmatrix} 0 & 0 \\ 0 & 8 \end{bmatrix} \quad (17)$$
$$\Delta_3 = \begin{bmatrix} 0 & -2 \\ 0 & 2 \end{bmatrix}; \quad \Delta_3 \Delta_3^H = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$$

Substituting the codeword matrix obtained into (15), (16) and (17) into (13) and (14), we can obtain the closed form expression for different error event in both slow and fast fading channel.

C Numerical Result and Discussion

In this section we provide numerical results for the closed form PEP enumerated in the previous section. For our result, we assume that k = 2 and $n_r = 1$. Fig. 2 shows the PEP for slow fading channel for N=1, 2 and 3. The PEP is also plotted for fast fading channel for error event of length 1, 2 and 3 in Fig. 3. Comparing the two graphs i.e. Fig. 2 and 3, the PEP of fast fading channel is smaller. The PEP at N=2 is the worst case for slow fading channel whereas for fast fading channel, the PEP at N=1 is the worst case.

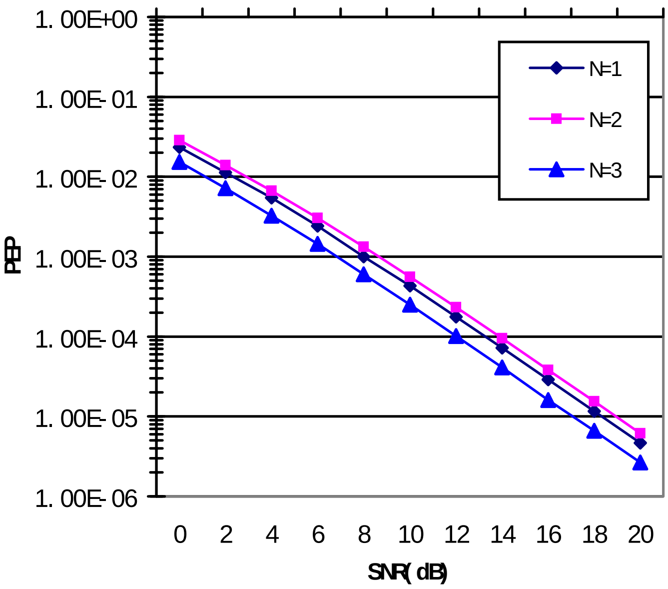

Fig 2. PEP performance of rate 1, 2-State BPSK HR-STTCM over quasi-static fading channel.

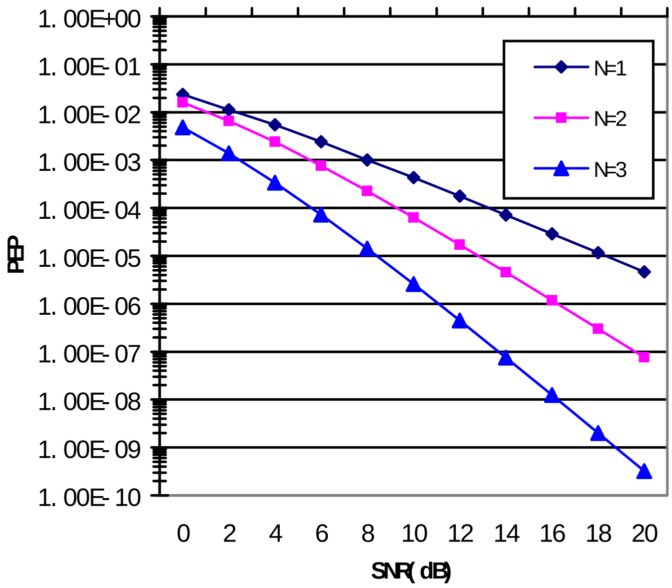

Fig. 3. PEP performance of rate 1, 2-State BPSK HR-STTCM over fast fading channel.

IV EVALUATION OF AVERAGE BIT ERROR PROBABILITY

In this section, we use the PEP's previously derived to evaluate in closed form an approximation to the average BEP, $P_b(E)$, based on accounting only for error events of length N.

Transfer function method [14] is a technique which makes use of code's state diagram to obtain error rate performance of trellis based codes. This method takes into account error event of all length. In [15] as estimation of bit error probability was obtained through accounting for error event paths of length to a pre-determined specific value using;

$$P_b(E) \approx \frac{1}{g} \sum_{X \neq \tilde{X}} q(X \to \tilde{X}) P(X \to \tilde{X}) \quad (18)$$

where g is the number of input bits per trellis transition and $q(X \to \tilde{X})$ is the number of bit errors associated with each error event.

Assuming transmission of the all zeros sequence, then for the 2-state HR-STTCM in Fig. 1, there is a single error event path of length 1, 4 error event paths of length 2 and 8 error event paths of length 3. The single error event has PEP_I obtain when N=1 whereas the 4 error event paths of length 2 all have PEP_{II} and the 8 error event paths of length 3 all have PEP_{III}. The PEP_I contribute one bit error whereas the four paths of PEP_{II} contribute a total of 12 bit errors and finally the 8 paths of PEP_{III} contribute a total of 28 bit errors. The average bit error probability when considering error event path of 1, 2 and 3 is given by P_{b1}, P_{b2} and P_{b3} respectively.

$$P_{b1} \approx \frac{1}{2}(PEP_I) \quad (19)$$

$$P_{b2} \approx \frac{1}{2}(PEP_I + 12 * PEP_{II}) \quad (20)$$

$$P_{b3} \approx \frac{1}{2}(PEP_I + 12 * PEP_{II} + 28 * PEP_{III}) \quad (21)$$

Fig. 4 and 5 show the average BEP of the code for fast and slow fading channels, respectively. In these figures, the approximate average BEP performances are plotted accounting for error event lengths up to 1 (19), (20) for 2, and (21) for 3. From the plot, we can observe that while considering error events length up to 2 is sufficient for calculating the average BEP for fast fading channels and error events with longer lengths are needed for estimating the average BEP over slow fading channel as can be seen by a comparison with the simulated results provided for the true BEP. Each frame consist of 256 bit in the simulated BEP. This slower convergence of the PEP to the average BEP as a function of the length of paths considering for slow fading relative to fast fading is consistent with a similar observation made [13] for orthogonal space time trellis codes

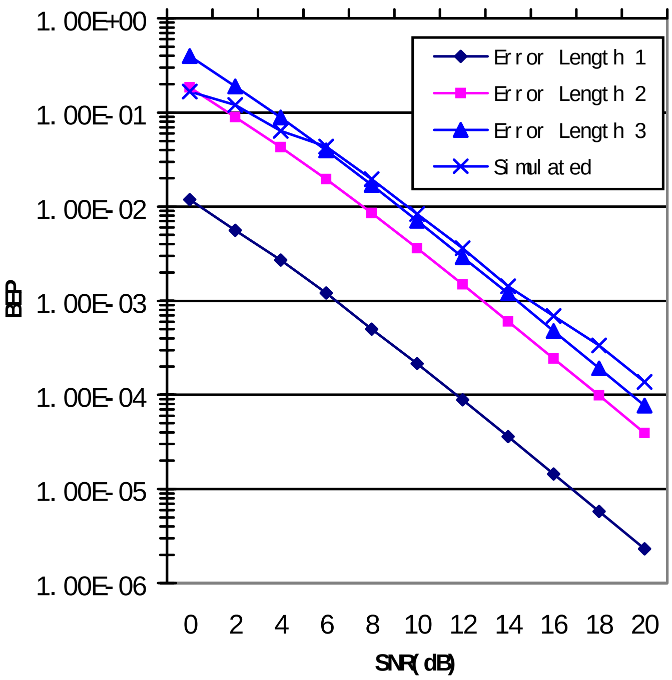

Fig 4. Average bit error probability of rate 1, 2-state BPSK HR-STTCM over quasi-static fading channel with one receive antenna.

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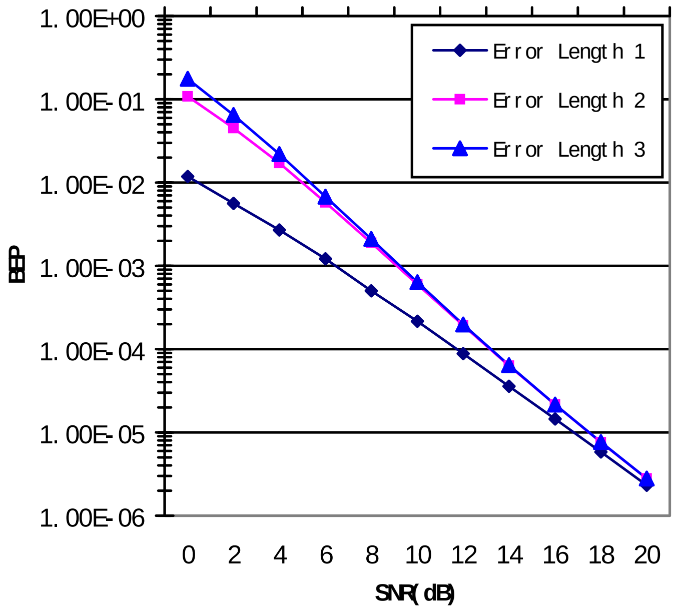

Fig. 5. Average bit error probability of rate 1, 2 -state BPSK HR-STTCM over fast fading channel with one receive antenna.

V. CONCLUSION

In this paper, the closed form expressions of the pairwise error probability of high rate space time trellis coded modulation using the orthogonal transmission matrix has been derived. We later, used the PEP obtained to estimate the average bit error probability. The Gauss-Chebyshev quadrature method used proved to be accurate and made the derivation of the closed form PEP easily obtainable. The method used here proved to be less complex than the others presented in the literatures.

APPENDIX

Here, the approximation in (11) is developed along with some error bounds. Consider the integral

$$I = \frac{1}{\pi}\int_0^{\infty} \frac{1}{t^2+1} \mathrm{f}\left(t^2+1\right) dt \qquad \text{(A.1)}$$

Substituting $y = 1/t^2 + 1$ into (A.1), (A.1) becomes;

$$I = \frac{1}{2\pi}\int_0^{1} \frac{1}{\sqrt{y(1-y)}} \mathrm{f}\left(1/y\right) dy \qquad \text{(A.2)}$$

The above equation (A.2) is of the form of an orthogonal polynomial in [12, 25.4.38] and Gauss-Chebyshev quadrature formula of first kind can be applied to solve it.

$$\int_{-1}^{1} \frac{\mathrm{f}\left(u\right)}{\sqrt{1-u^2}} du = \sum_{i=1}^{k} w_i \,\mathrm{f}\left(u_i\right) + \mathrm{R}_k \qquad \text{(A.3)}$$

$$u_i = \cos\frac{(2i-1)\pi}{2k}$$

$$w_i = \frac{\pi}{k}$$

$$\mathrm{R}_k \le \max_{-1<x<+1} \frac{\pi}{(2k)!2^{2k-1}} \left| \mathrm{f}^{2k}(u) \right|$$

The expression in (A.2) can be reduce to (A.3) if we express $2y-1=u$

$$\begin{aligned} 2y-1 &= \cos\frac{(2i-1)\pi}{2k} \\ 2y &= \cos\frac{(2i-1)\pi}{2k}+1 \end{aligned} \tag{A.4}$$

Using trigonometric function in (A.5), y can be expressed as (A.6).

$$\cos m = \cos\left(\frac{m}{2}+\frac{m}{2}\right) = \cos^2\frac{m}{2} - \sin^2\frac{m}{2} \tag{A.5}$$

$$\begin{aligned} y &= \cos^2\frac{(2i-1)\pi}{4k} \\ 1/y &= \sec^2\frac{(2i-1)\pi}{4k} \end{aligned} \tag{A.6}$$

Accordingly one has that

$$I = \sum_{i=1}^{k} w_i\,\mathrm{f}(u_i) = \frac{1}{2k}\sum_{i=1}^{k}\mathrm{f}\left(\sec^2\frac{(2i-1)\pi}{4k}\right) \tag{A.7}$$

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