

Low Complexity Adaptive Receivers for Joint Equalization and Interference Cancellation in Space-Time Block-Coded Systems

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Abstract — This paper describes the low-complexity adaptive receiver, based on the recursive least squares (RLS) algorithm, that has been designed for space-time block-coded (STBC) transmissions over frequency – selective fading channels with rapid variations [7]. The space-time block coded structure is exploited to reduce the complexity of the RLS algorithm to that of a least mean squares (LMS) algorithm. The performance of the adaptive receiver is compared to the system with perfect channel state information (CSI). In addition, the effect different parameters in the RLS algorithm are provided in the simulation results.

Index Terms — Frequency-selective channel, space-time block codes, adaptive receiver, MIMO, EDGE.

I. INTRODUCTION

NEXT generation broadband wireless channels are expected to provide users with high-speed data transmission capabilities to bring the dreams of wireless internet access, multimedia services and mobile computing features much closer to reality. Increasing the system capacity without requiring additional power and bandwidth has become a major factor to ensure the availability of the scarce spectral resource.

In this respect, multiple-input multiple-output (MIMO) systems have been proven to significantly increase the capacity in rich scattering environments. In particular, STBCs, which were first introduced in [1] and later generalized in [2], have been an attractive MIMO technique due to its simple linear processing and lack of additional bandwidth and transmission power requirements. The Alamouti scheme has in-fact been adopted in WCDMA and CDMA2000 standards.

However, broadband wireless channels introduce random attenuations and delays to the transmitted signals and, due to the increased data rates, cause inter-symbol interference (ISI) at the receiver. Such channels are termed frequency-selective broadband channels. The Alamouti scheme was design for narrow band flat fading channels. The first extensions of STBCs was developed in [3] with the so-called time-reversal (TR) STBC. The third generation TDMA cellular standard is known as *Enhanced Data Rates for Global Evolution* (EDGE – also referred to as 2.5G). EDGE results in a challenging equalization problem due to the use of 8-PSK modulation and general non minimum-phase characteristics of the typical urban (TU) channel and additional ISI due to the Gaussian minimum shift keying (GMSK). To cater for the “EDGE effect”, the single carrier frequency domain equalizer (SC-FDE) schemes with zero post-fix [4] and cyclic prefix [5] were developed. These schemes were later generalized in [6], which bench marks STBCs in frequency-selective channels. The SC-FDE has two main advantages over the orthogonal frequency division multiplexing (OFDM) counterpart, namely, lower peak-to-average ratio (PAR) and reduced sensitivity to carrier frequency errors.

All the above mentioned receivers require channel state information (CSI) at the receiver. One approach is to use training sequences embedded in each block to estimate the CSI. This, however, results in increased system overhead. In addition to this, the mobility of the users may cause the channel impulse response to vary rapidly and hence the quasi-static assumption of the channel becomes void. The use of longer blocks, which is required to reduce the system overhead in such training based schemes, can not be practical in such cases. This motivated the development of an effective low-complexity adaptive receiver with fast tracking abilities [7].

The results provided in [7] showed the performance of the algorithm for various Doppler frequencies. But the question still left to be answered is “how well does the system perform as compared to a system with perfect CSI?” Answering this question will quantify the actual tracking performance of the adaptive receiver. Motivated by this, we provide simulation results and compare the results to the system with perfect CSI.

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Furthermore, an indication of how various parameters can be adjusted to achieve near “perfect CSI performance” can be determined.

The rest of this paper is organized as follows. In section II, the cyclic prefix (CP) transmission required to eliminate the inter-symbol interference (ISI) effect and induce a circulant channel structure, is discussed. The circulant structure can be exploited by the receiver to great advantage as will be shown in the rest of the paper. Section III details the SC-FDE and decoding designs in an MMSE sense. Section IV describes the adaptive receiver for training and tracking mode developed in [7]. The simulation results are provided in section V, while conclusions are drawn in Section VI.

Notation: Upper case letters denote matrices, lower case letters stand for column vectors; $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ represent conjugate, transpose and hermitian, respectively; I_K denotes an identity matrix of size I_K . F stands for a $N \times N$ Fast Fourier Transform (FFT) matrix.

II. CYCLIC-PREFIX TRANSMISSION

The long impulse response sequences associated with frequency selective channels results in inter-symbol interference (ISI) and hence degrade the system performance. To combat the ISI effects, equalization and extra symbols, known as cyclic prefix (CP), are used to decompose the frequency selective channel into independent flat channels. This reduces the required complexity of the equalizer.

Consider the data sequence $x(n)$ that is transmitted over a frequency selective channel $h = (h(0) \cdots h(v))^T$, where v is the memory of the channel. The received sequence $y(n)$ is given by the convolution sum

$$y(n) = \sum_{k=0}^v h(k)x(n-k) + w(n) \quad (1)$$

where $w(n)$ noise samples are assume zero mean Gaussian with white power spectrum. The above expression clearly shows that the received symbol at time n is a linear combination of $v+1$ transmitted symbols which is the ISI effect. In order to combat ISI, the frequency- selective channel h is converted into parallel flat channels. By collecting N samples of the received sequence and ignoring the noise samples, (1) can be re-written as

$$\begin{pmatrix} y(0) \\ \vdots \\ y(N-2) \\ y(N-1) \end{pmatrix} = \begin{pmatrix} h(v) & \cdots & h(0) & & & \\ & \ddots & \vdots & \ddots & & \\ & & h(v) & \cdots & h(0) & \\ & & & h(v) & \cdots & h(0) \end{pmatrix} \begin{pmatrix} x(-v) \\ \vdots \\ x(-1) \\ x(0) \\ \vdots \\ x(N-2) \\ x(N-1) \end{pmatrix} \quad (2)$$

The channel matrix is toeplitz in structure of dimensions $N \times (N+v)$. If the symbols $\{x(-v), \dots, x(-1)\}$ are replaced by $\{x(N-v), \dots, x(N-1)\}$, then (2) reduces to

$$\begin{pmatrix} y(0) \\ \vdots \\ y(N-2) \\ y(N-1) \end{pmatrix} = \begin{pmatrix} h(0) & 0 & \cdots & h(v) & \cdots & h(0) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & h(v) & \cdots & h(0) & 0 \\ 0 & 0 & 0 & h(v) & \cdots & h(0) \end{pmatrix} \begin{pmatrix} x(0) \\ \vdots \\ x(N-2) \\ x(N-1) \end{pmatrix} \quad (3)$$

The important result to be observed is that the channel matrix is now $N \times N$ circulant matrix. Defining the above channel matrix as \tilde{H} , this implies \tilde{H} can be diagonalized through the following decomposition

$$\tilde{H} = F^* \Lambda F \quad (4)$$

where F is the $N \times N$ unitary discrete Fourier transform (DFT) matrix. If we define $\mathbf{y} \triangleq Fy$ and $\mathbf{x} \triangleq Fx$, then, by applying the DFT matrix to the received data block in (3), we get

$$\begin{pmatrix} \mathbf{y}(0) \\ \vdots \\ \mathbf{y}(N-2) \\ \mathbf{y}(N-1) \end{pmatrix} = \begin{pmatrix} \Lambda(0) & & & \\ & \ddots & & \\ & & \Lambda(N-1) & \end{pmatrix} \begin{pmatrix} \mathbf{x}(0) \\ \vdots \\ \mathbf{x}(N-2) \\ \mathbf{x}(N-1) \end{pmatrix} \quad (5)$$

Hence the original sequence $\{y(n)\}$ in (3) suffered from the ISI effect is now decomposed into a sequence in which each entry is equivalent to transmitting symbols over a flat channel $\Lambda(n)$.

III. SINGLE-USER TRANSMISSION

In the description to follow, it is assumed that each user is equipped with two transmit antennas and the receiver has a single antenna. Let the N -symbol block including CP transmitted from the first and second antennas at block time k

be given by x_1 and x_2 , respectively. At block time $k+1$, permuted conjugate versions $-Px_2^*$ and Px_1^* are sent from the first and second antennas, respectively. The permutation matrix P is a circular reversal matrix given by

$$P = \begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & \ddots & \\ & & & 1 \end{pmatrix} \quad (6)$$

Note that this is a special case of the permutation matrix described in [6]. Hence the received blocks k and $k+1$ in the presence of noise is given by

$$y^{(i)} = H_1^{(i)}x_1^{(i)} + H_2^{(i)}x_2^{(i)} + w^{(i)} \quad \text{for } i = k, k+1 \quad (7)$$

where $w^{(i)}$ is the noise term which assume zero mean Gaussian with white power spectrum, and $H_1^{(i)}$ and $H_2^{(i)}$ are the circulant channel matrices from antenna one and two, respectively, to the receive antenna. The circulant channel results from the addition of the cyclic prefix of length ν described in section 1.

Next, applying the $N \times N$ DFT matrix F to the received sequence, we get

$$Y^{(i)} \triangleq Fy^{(i)} = \Lambda_1^{(j)}X_1^{(j)} + \Lambda_2^{(j)}X_2^{(j)} + W^{(j)} \quad (8)$$

where $X^{(i)} \triangleq Fx^{(i)}$, $W^{(i)} \triangleq Fw^{(i)}$ and $\Lambda_1^{(i)}$ and $\Lambda_2^{(i)}$ are the diagonal matrices of H_1 and H_2 , respectively. Using the properties of DFT the terms for the $k+1$ block can be written as

$$\begin{aligned} X_1^{(k+1)}(n) &= -X_2^{*(k)}(n) \\ X_2^{(k+1)}(n) &= X_1^{*(k)}(n) \end{aligned} \quad (9)$$

Combining (8) and (9), we arrive with

$$Y = \begin{pmatrix} Y^{(k)} \\ Y^{*(k+1)} \end{pmatrix} = \begin{pmatrix} \Lambda_1 & \Lambda_2 \\ \Lambda_2^* & -\Lambda_1^* \end{pmatrix} \begin{pmatrix} X_1^{(k)} \\ X_2^{(k)} \end{pmatrix} + \begin{pmatrix} W^{(k)} \\ W^{*(k+1)} \end{pmatrix} \quad (10)$$

It should be noted that the choice of the permutation matrix (6) alleviates the need of post multiplying the received sequence by the P as in [6]. It is clear, from the Alamouti-like diagonal channel matrix in (10), that by forming the unitary matrix

$$\Lambda^H \triangleq \begin{pmatrix} \Lambda_1 & \Lambda_2 \\ \Lambda_2^* & -\Lambda_1^* \end{pmatrix}^H$$

the data sequence can be decoupled by the following unitary operation

$$\tilde{Y} \triangleq \Lambda^* Y = \begin{pmatrix} \Lambda_0 & 0 \\ 0 & \Lambda_0 \end{pmatrix} \begin{pmatrix} X_1^{(k)} \\ X_2^{(k)} \end{pmatrix} + \tilde{W} \quad (11)$$

where $\Lambda_0 \triangleq (\Lambda_1\Lambda_1^* + \Lambda_2\Lambda_2^*)^{1/2}$. The minimum mean square estimator (MMSE) of X is then given as

$$\hat{X} = \tilde{\Lambda} \Lambda^* Y = \left(\Lambda^* \Lambda + \frac{1}{\text{SNR}} \mathbf{I}_{2N} \right)^{-1} \Lambda^* Y \quad (12)$$

where $\tilde{\Lambda}$ is the diagonal scaling matrix and SNR is the signal-to-noise at the receiver given by $\text{SNR} = \sigma_x^2 / \sigma_w^2$.

IV. ADAPTIVE SYSTEM

The interference cancellation and equalization technique described in the previous section requires the knowledge of the channel state information (CSI) at the receiver. This is usually accomplished by the addition of training (pilot) sequence to each transmitted block in order to estimate the channel, at the expense of addition bandwidth requirement. In order to decrease the overhead requirements, such pilot aided schemes will require longer blocks which become impractical in channels with fast variations.

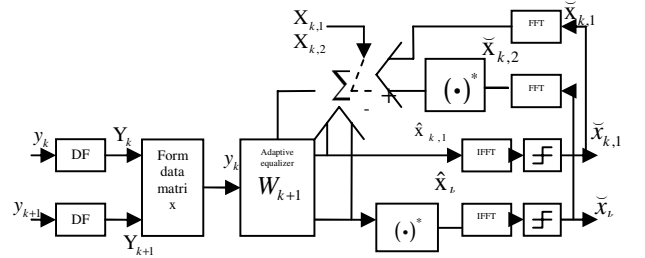


Fig.1 Block diagram for the single user adaptive receiver with two-transmit one-receive antennas.

Hence the need of an adaptive receiver under such conditions becomes imperative. The RLS algorithm provides fast tracking and, due to the special structure of space-time block codes, the complexity can be reduce to that of a LMS algorithm.

Defining the combined matrix of the MMSE in (12) as $B \triangleq \tilde{\Lambda} \Lambda^*$, it can be shown that this matrix has an Alamouti structure i.e.

$$B = \begin{pmatrix} B_1 & B_2 \\ B_2^* & -B_1^* \end{pmatrix}$$

where

$$\begin{aligned} \mathbf{B}_1 &= \text{diag} \left\{ \frac{1}{\Lambda_0(i,i)+1/\text{SNR}} \right\}_{i=0}^{N-1} \cdot \Lambda_1^* \\ \mathbf{B}_2 &= \text{diag} \left\{ \frac{1}{\Lambda_0(i,i)+1/\text{SNR}} \right\}_{i=0}^{N-1} \cdot \Lambda_2 \end{aligned} \quad (13)$$

Hence (12) can be written as

$$\begin{pmatrix} \hat{\mathbf{X}}_1^{(k)} \\ \hat{\mathbf{X}}_2^{(k)} \end{pmatrix} = \begin{pmatrix} \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{B}_2^* & -\mathbf{B}_1^* \end{pmatrix} \mathbf{Y} \quad (14)$$

This can be rearranged into the following form.

$$\begin{pmatrix} \hat{\mathbf{X}}_1^{(k)} \\ \hat{\mathbf{X}}_2^{(k)} \end{pmatrix} = \begin{pmatrix} \text{diag}(\mathbf{Y}^{(k)}) & \text{diag}(\mathbf{Y}^{*(k)}) \\ -\text{diag}(\mathbf{Y}^{(k+1)}) & \text{diag}(\mathbf{Y}^{*(k+1)}) \end{pmatrix} \begin{pmatrix} \mathbf{E}_1 \\ \mathbf{E}_2 \end{pmatrix} \quad (15)$$

$\hat{\mathbf{X}} \triangleq \mathbf{U}_k \mathbf{E}$

The vectors \mathbf{E}_1 and \mathbf{E}_2 contain the diagonal entries of \mathbf{B}_1 and \mathbf{B}_2 , respectively. The equalizer coefficients \mathbf{E} are adaptively calculated using the following recursion for every two blocks.

$$\mathbf{E}_{k+2} = \mathbf{E}_k + \mathbf{Q}_{k+2} \mathbf{U}_{k+2}^* [\mathbf{D}_{k+2} - \mathbf{U}_{k+2} \mathbf{E}_k] \quad (16)$$

where

$$\begin{aligned} \mathbf{Q}_{k+2} &= \lambda^{-1} [\mathbf{Q}_k - \lambda^{-1} \mathbf{Q}_k \mathbf{U}_{k+2} \\ &\times (\mathbf{I}_{2N} + \lambda^{-1} \mathbf{U}_{k+2} \mathbf{Q}_k \mathbf{U}_{k+2}^H)^{-1} \mathbf{U}_{k+2}^H \mathbf{Q}_k] \end{aligned} \quad (17)$$

The parameters of the algorithm are initialized as follows: $\mathbf{E}_0 = 0$ and $\mathbf{Q}_0 = \delta \mathbf{I}_{2N}$, with δ being a large number. λ is called the forgetting factor and can be in the range $\{0,1\}$. Using a smaller forgetting factor will result in faster tracking of the RLS algorithm. However this may result in numerical problems due the accuracy required in the algorithm. It therefore becomes obvious that careful selection of λ is required and is further discussed in the next section.

\mathbf{D}_k is referred to as the desired response and is given by,

$$\mathbf{D}_{k+2} = \begin{cases} \begin{pmatrix} \mathbf{X}_1^{(k+2)} \\ \mathbf{X}_2^{*(k+2)} \end{pmatrix}, & \text{for training} \\ \begin{pmatrix} \tilde{\mathbf{X}}_1^{(k+2)} \\ \tilde{\mathbf{X}}_2^{*(k+2)} \end{pmatrix}, & \text{for decision-direct tracking} \end{cases} \quad (18)$$

\mathbf{Q}_{k+2} has the following diagonal structure.

$$\mathbf{Q}_{k+2} = \begin{pmatrix} \mathbf{Q}_{k+2} & 0 \\ 0 & \mathbf{Q}_{k+2} \end{pmatrix} \quad (19)$$

It is easy to see that the inverse term in (17) is given by a diagonal matrix

$$(\mathbf{I}_{2N} + \lambda^{-1} \mathbf{U}_{k+2} \mathbf{Q}_k \mathbf{U}_{k+2}^H)^{-1} = \begin{pmatrix} \Theta_{k+2} & 0 \\ 0 & \Theta_{k+2} \end{pmatrix} \quad (20)$$

The diagonal matrix Θ_{k+2} is given by

$$\Theta_{k+2} = \left(\mathbf{I}_N + \lambda^{-1} \text{diag} \left(|\mathbf{Y}^{(k)}|^2 + |\mathbf{Y}^{(k+1)}|^2 \right) \right)^{-1} \quad (21)$$

This results in \mathbf{Q}_{k+2} having the following structure.

$$\mathbf{Q}_{k+2} = \lambda^{-1} (\mathbf{Q}_k - \lambda^{-1} \mathbf{Q}_k \Omega_{k+2} \mathbf{Q}_k) \quad (22)$$

where

$$\Omega_{k+2} = \Theta_{k+2} \text{diag} \left(|\mathbf{Y}^{(k)}|^2 + |\mathbf{Y}^{(k+1)}|^2 \right) \quad (23)$$

Hence we get

$$\Omega_{k+2} = \left[\text{diag} \left(|\mathbf{Y}^{(k)}|^2 + |\mathbf{Y}^{(k+1)}|^2 \right) + \lambda^{-1} \mathbf{Q}_k \right]^{-1} \quad (24)$$

Finally the RLS equalizer is then given by

$$\mathbf{E}_{k+2} = \mathbf{E}_k + \begin{pmatrix} \mathbf{Q}_{k+2} & 0 \\ 0 & \mathbf{Q}_{k+2} \end{pmatrix} \mathbf{U}_{k+2}^* [\mathbf{D}_{k+2} - \mathbf{U}_{k+2} \mathbf{E}_k] \quad (25)$$

At first glance the RLS algorithm may appear computationally complex, due to the number of matrix inversions required in the algorithm. However, due to the diagonal structure that resulted from the special structure of the STBCs, the matrix inversions are in fact scalar inversions. This results in an LMS complexity. The overall adaptive receiver is shown in Figure 1.

V. SIMULATION RESULTS

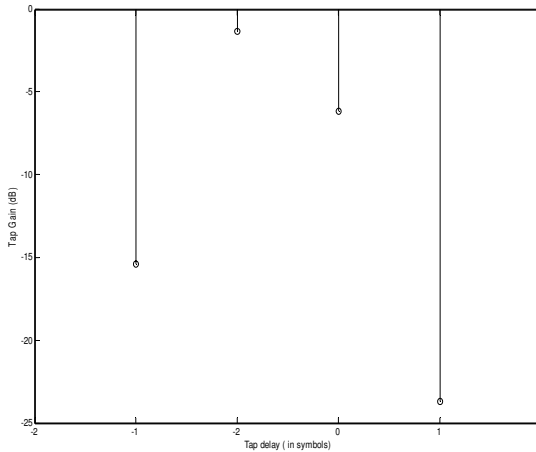


Fig.2 Equivalent CIR for a TU channel with GMSK pulse shaping

Table 1 A typical urban (TU) channel model

Delay (μsec)	0.0	0.2	0.5	1.6	2.3	
Strength (dB)	-3.0	0.0	-2.0	-6.0	-8.0	-10

In the simulations, we assume a symbol rate of 271 kSymbols/s. A typical urban (TU) channel, with the power-delay profile shown in Table 1, is used along with a linearized Gaussian minimum shift keying (GMSK) transmit pulse shape. The average profile of the overall channel impulse response is shown in Fig 2. From the figure, the channel memory is $\nu = 3$. All channels are assumed independent.

Being consistent with the EGDE standard, each user has two transmit antennas with 8-PSK constellation. A Data block size of 32 symbols plus three cyclic prefix symbols is used. The algorithm is re-trained after 50 data blocks. The forgetting factor is set to 0.95.

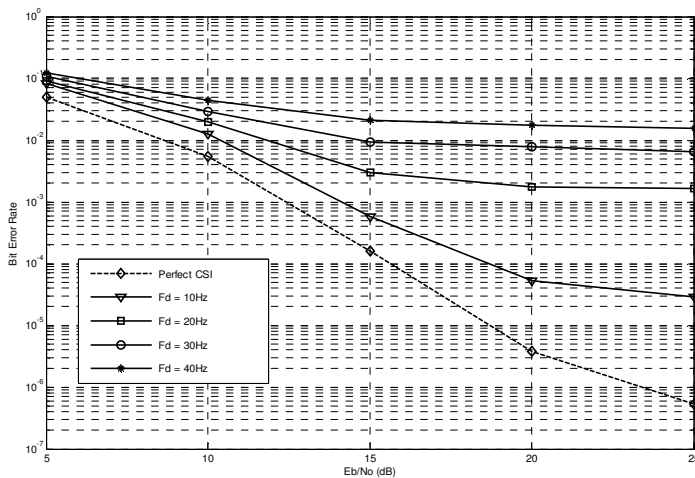


Fig.3 Effect of Doppler frequency on the performance of RLS algorithm.

Fig. 3 shows the performance of the RLS algorithm at different Doppler frequencies as compared to the system with perfect CSI. From the figure it is clear that the performance of the algorithm approaches that of perfect CSI as the Doppler frequency decreases. This shows that the algorithm suffers in channels with faster variations.

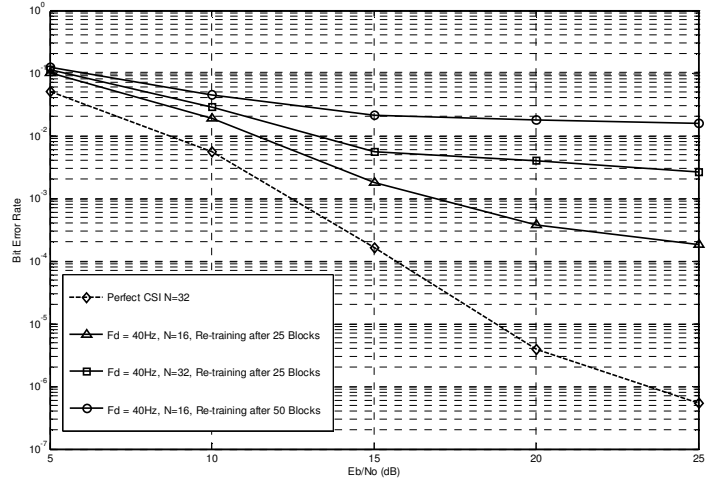


Fig.4 Effect of block size and re-training rate

By decreasing the block length and increasing the rate at which re-training is done, the performance of the adaptive receiver is improved drastically as shown in Fig 4. With the Doppler frequency set to 40 Hz, it can be seen that the performance approaches that of the system with perfect CSI as the training rate is increased and the block size is decreased. It can be concluded that at higher channel variations a smaller block size with a higher re-training frequency is required.

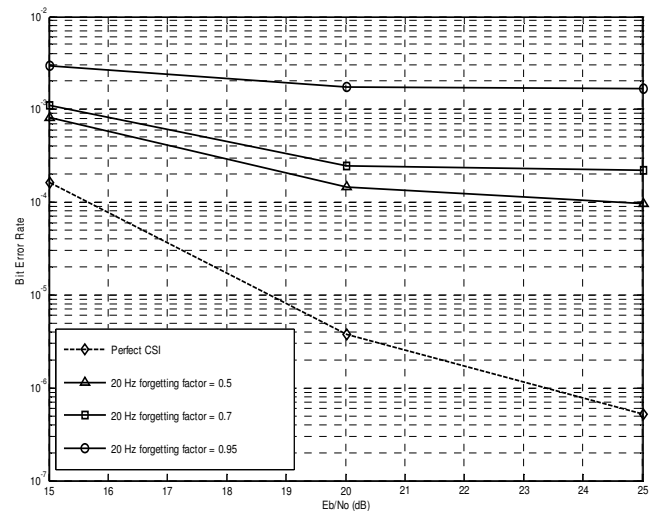


Fig.5 Effect of forgetting factor

Fig.5 shows the effect of varying the forgetting factor, λ . Decreasing λ results in improved performance at the expense of additional complexity of the algorithm. For the simulation, the block size and Doppler frequency was set to 32 symbols and 20 Hz, respectively. λ was varied between the values of

0.95, 0.7 and 0.5. It can be seen that there is a significant improvement when varying λ from 0.95 to 0.7. However the change in performance decreases significantly between 0.7 and 0.5. Hence, when selecting λ , there is a tradeoff between the performance improvement required and the complexity that is introduced by decreasing λ .

V. CONCLUSION

The adaptive receiver for space-time block-coded transmissions has been discussed for the single user case. The receiver does not require CSI. By using the system with perfect CSI as a benchmark, a method of optimizing various parameters such as block size, re-training frequency and forgetting factor has been shown. Therefore, depending on the channel variations present and performance required, values for these parameters can be selected.

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