Experimental models for network mesh topology with designs that enhance survivability*

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Abstract **- Network Design problems involving survivability usually include tradeoff of the potential for lost revenues and customer goodwill against the extra costs required to increase the network survivability. It also involves selection of nodes and edges from lists of potential sets to accomplish certain desirable properties. In many applications it is imperative to have built-in survivability of the network. Delays of traffic are undesirable since it affects QoS to clients of the network. An optimization system with survivability was constructed and explained in this paper that may help in the planning of mesh topologies and link capacities to avoid costly designs while maintaining a certain degree of survivability of the network. Experimental models are formulated by using integer programming techniques and the solution approach is based on exact optimization methods. The main benefit of this system is that a network designer can zoom into a part of a larger network that interests him or her and provide a survivable design for that part.**

Index Terms - **Survivable network design, connectivity, survivability, integer programming**.

I. Introduction

The design of node independent and edge independent paths through a connected network problem is a fundamental problem in network mesh topology design. This problem arises in the design of communication networks that have resilience to single-edge and single-node failures and is an important special case in the design of survivable networks [5] and [10].

In such networks, there are at least two edge-disjoint paths between certain pairs of nodes. So if an edge fails, it is always in principle possible to reroute the traffic between a source-destination pair along the second path. This problem is a particular case of the two-connected networks with bounded meshes problem studied by Fortz, Labbe, and Maffioli [3].

In designing a mesh network with survivability, topology optimization is clearly difficult and even sub-problems like routing and capacity assignment have been shown to be difficult, Kershenbaum [6]. Integer programming techniques have been used to produce optimal solutions or tight bounds on optimal solutions only for problems of a few dozen nodes, Gavish [4]. For problems of practical size, virtually all approaches have been heuristics.

Work on methods for designing survivable communication networks by Martin Grotschel *et al.* [5] concludes that "two-connected" topologies provide a high level of survivability in a cost effective manner, and that it is feasible to compute minimum cost networks that satisfy survivability for small problems. For large problems one often has to be satisfied with approximately optimal solutions.

For our work, as the title suggests, we have focused on experimental models and investigated the feasibility of designing survivable networks by using exact optimization methods. Our purpose is to determine a network topology that should provide for at least two diverse paths between certain "special" offices (nodes), thus providing for protection against any single link or single node failure for traffic between these offices. The system that we have developed has proven effective in networks with up to 15 nodes.

This paper is organized as follows. In Section II, we briefly introduce some network models and describe the survivable model. Section III describes the system development aspects and models incorporated. In Section IV, we give an illustrative example for 2ECON and 2NCON. In section V, we present some experimental results. Section VI, describes the conclusion. Finally we conclude with section VII that envisages future work.

II. Network Designs

A. Capacitated network models

Consider a graph $G = (V, A)$ where V is the set of nodes (vertices) and A the set of (potential) edges. We assume that $n = |V|$. In the basic capacitated form, it is assumed that when an edge *ij* is introduced to form part of the design, the total flow of commodities must be smaller than or equal to the total capacity of that specific edge *ij* (see equation (3) below). For the lower bounds we must similarly ensure that the flow of each commodity on an edge *ij* is larger than the lower bound for that edge (if necessary), if the edge is part of the design (see equation (4) below). We denote these upper and lower bounds by u_{ij}^k and l_{ij}^k

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where the superscript k refers to the commodity being transmitted. We further assume that we need to route multiple commodities (K in number) on the network and that each commodity k has (as a first simplification) a single source node s^k and a single destination d^k. If x^k denotes the vector of flows of commodity k on the network, we will assume that the elements x^k viz x^k_{ij} denote the fraction of the required flow commodity k to be routed from s^k to d^k that flows on an edge *ij*. We let c^k denote the cost vector for commodity k, which we scale to reflect the definition of x^k_{ij}. Also we let y be a zero-one vector with elements y_{ij} indicating whether an edge *ij* is selected as part of the network design. If similarly f is the vector of fixed costs (installation costs of edge *ij*) the problem becomes:

$$\text{Minimize } \sum_{1\le k\le K} c^k x^k + fy \quad (1)$$

subject to

$$\sum_{\{j:ij\in A\}} x^k_{ij} - \sum_{\{j:ji\in A\}} x^k_{ji} = \begin{cases} 1 & \text{if } i=s^k \\ -1 & \text{if } i=d^k \\ 0 & \text{otherwise} \end{cases} \quad \forall i\in N, k=1,2,...,K, \quad (2)$$

$$\sum_{1\le k\le K} \frac{1}{u^k_{ij}} x^k_{ij} \le y_{ij} \quad \forall ij\in A, k=1,2,..,K \quad (3)$$

$$l^k_{ij} y^k_{ij} \le x^k{}_{ij} \quad \forall ij\in A, k=1,2,..K, \quad (4)$$

$$x^k_{ij} \ge 0 \quad \forall ij\in A, k=1,2,..K, \quad (5)$$

$$y_{ij} = 0 \; or \; 1 \quad \forall ij\in A \quad (6)$$

The constraints (3) and (4) are often called "forcing constraints". If the forcing constraints are removed from the capacitated problem, the resulting problem decomposes into a number of independent shortest path problems.

B. Models with discrete flow cost alternatives

In many real life situations the cost of flow along an edge *ij* is given by a discrete function and not a continuous function since additional capacity has to be installed if flow exceeds a certain level. This model is indicated in cases where certain facility "lines" (e.g. *T0, T1* and *T2*) can be introduced at a cost (design cost) to enable the network to handle the increased flow more economically.

In papers by Leung *et al*. [8] and Magnanti *et al*. [9] such a design problem is discussed. These models arise in problems such as the following:

In designing telecommunication networks, we would like to install sufficient capacity to carry required traffic (telephone calls, data transmissions) simultaneously between various source-sink locations. Suppose that (s^k, d^k) for $1 \le k \le K$ denote K pairs of source-sink locations and x^k denotes the volume of messages sent from the source s^k to sink d^k. We assume that we can install either or both of two different types of facilities on each link of the transmission network, so-called *T0* and *T1* lines. Each *T0* line can carry some units of messages and each *T1* line can carry more units of messages; installing a *T0* line on an edge *ij* incurs a cost of a_{ij} and installing a *T1* line on an edge *ij* incurs a cost of b_{ij}. Once we have installed the lines we incur no additional costs in sending flow on them. This problem arises in practice because companies with large telecommunication requirements might be able to lease lines more cost-effectively than paying public tariffs.

C. Model considered for empirical work and system (Survivable-Netdesigner) development

Before we consider the model, we have to introduce certain concepts and notation for survivability issues. We follow the notation in Grotschel *et al*. [5] for these purposes. The problem of designing survivable mesh networks can be modelled as a minimum cost network design problem with certain low-connectivity constraints. We are given a *graph* $G = (V, A)$, where V is a set of *nodes* that represent offices that must be interconnected by a network, and A is a collection of *edges* that represent the possible pairs of nodes between which direct transmission links can be placed.

The *survivability conditions* require that the network satisfy certain edge and node connectivity requirements. In particular, a nonnegative integer r_s is associated with each node $s \in V$ that represents its *connectivity requirement*. For our purposes we implicitly assume that r is a vector of node connectivity types with $r \in \{0, 1, 2\}^n$.

For each pair of distinct nodes $s, t \in V$, the network $N = (V, F)$ where $F \subseteq A$ to be designed has to contain at least $r(s, t) := \min\{r_s, r_t\}$ edge-disjoint (or node-disjoint) [*s*, *t*] paths.

We define the concept of a cut (induced by $W \subseteq V$) as $\delta(W) := \{ij \in A \mid i \in W, j \in V \setminus W\}$ or $\delta_G(W)$ to make it clear that the graph G is referred to.

We extend the connectivity requirement function *r* to functions operating on sets by setting:

$$r(W) := \max\{r_s \mid s \in W\} \; \forall W \subseteq V \text{ and}$$

$$con(W) := \max\{r(s,t) \mid s \in W, t \in V \setminus W\}$$

$$\forall W \subseteq V \text{ and } \phi \ne W \subset G.$$

For any subset of links $F \subseteq A$, we define

$$y(F) := \sum_{ij\in F} y_{ij}.$$

$$\text{Where } y_{ij} = \begin{Bmatrix} 1 & if \quad ij \in F \\ 0 & otherwise \end{Bmatrix}$$

For our purposes we decided as a first step to concentrate on the network topology ignoring edge (arc or link) capacities and concentrating on survivability issues of the network.

The special case we investigated seems to be important for practical use. See also Cardwell *et al*. [1], and Monma and Shallcross [10], where they address the problem of designing survivable fiber optic networks. We follow the notation of Grotschel *et al*. [5], and define 2ECON (and 2NCON) problems where edge-disjoint (respectively node-disjoint) paths are required.

Formally a 2ECON or 2NCON problem is given by (G, r) where we implicitly assume that $G = (V, A)$ is a graph and r a vector of node types with $r \in \{0, 1, 2\}^n$ as mentioned before. Grotschel *et al.* [5] then considered a model where the objective only considers fixed costs c_{ij} if edge *ij* is used in the design.

The model becomes:

$$\text{Minimize} \quad \sum_{ij \in A} c_{ij} y_{ij}$$

Subject to

$$y(\delta(W)) \geq con(W) \qquad \forall W \subseteq V, \; \phi \neq W \neq V; \tag{8}$$

$$y(\delta_{G-z}(W)) \geq 1 \qquad \forall z \in V, and \; \forall W \subseteq V \setminus z, \; \phi \neq W \neq V \setminus z \; with \; r(W) = 2 \; and \; r(V \setminus (W \cup z)) = 2; \tag{9}$$

$$y_{ij} \in \{0,1\} \qquad \forall ij \in A. \tag{10}$$

The y_{ij} denote variables that take on values $1 \; (or \; 0)$ for the case where edge *ij* is used (not used) in the design. It follows from Menger's theorem that, for every feasible solution X of (8), (9) and (10), the subgraph $N = (V, F)$ of G defines a network satisfying the two-connected node survivability requirements. Menger's theorem is a basic result about connectivity in finite undirected graphs.

According to Wikipedia [12], Menger's theorem states that given a finite undirected graph G and two nonadjacent nodes i and j, the size of the minimum node cut for node i and node j (the minimum number of nodes whose removal disconnects node i and node j) is equal to the maximum number of pair-wise node-independent paths from node i to node j.

The solution to this model ignoring constraints (9) gives rise to a so-called 2ECON problem that protects edge independent paths for certain node pairs. Similarly they use the notation of 2NCON for problems where node independence of paths between certain high-connectivity nodes also needs to be protected. For 2NCON problems, constraints (9) must be included in the model.

III. System Development Aspects

A. *Software development aspects*

A survivable network design system was developed in Qt, a multiplatform C++ application development framework. One source runs natively on four different platforms (Windows, Unix/Linux, Mac OS X, and embedded Linux). The software can be ported to multiple platforms with a simple recompile. The system capabilities include the selection of an optimal link and node topology from a given potential network. This selection can be done such that the resulting network is 2ECON or 2NCON survivable (or both) for any pair of nodes with connectivity 2 (and the others with connectivity 1). The system output is a plain text Integer Linear Programming (ILP) problem that can be transferred to a machine capable of solving integer programming problems [7].

Experiments with several survivable network design instances could be solved to the (exact) optimum in problems with up to 15 nodes.

B. *Survivable-netdesigner system specifications and built-in capabilities*

The survivable-network designer's properties and functions are described below

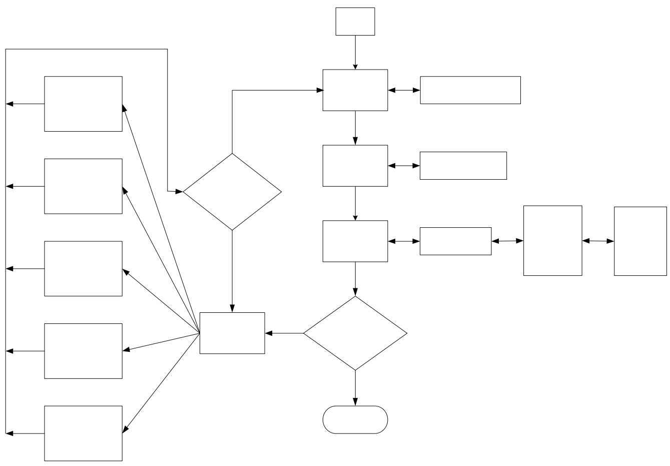

Figure 1 Flow chart of network designer system

1) Open or Create a Network

An existing network can be opened and altered or a new network can be constructed.

2) Adding a Potential node

A new potential (transhipment) node can be added to or removed from the network. A node may be assigned a design cost or the design cost can be set equal to zero.

3) Adding a Potential Link

Potential links joining node pairs in the network may also be added to or removed from the network design. Three types of potential links may be included. These potential links can be assigned different design costs. The design cost is the cost associated for establishing the type of link between the node pairs.

4) Editing Nodes

The design cost of a node may be edited or a node may be disabled (excluded from the potential configuration).

5) Edit Potential links

The design cost of a specific link may be altered. Potential links may also be disabled (excluded from potential configuration if one wishes to study the effect of a link failure)

6) Construct ILP

The current network design problem is modelled with mathematical modelling techniques as an optimization problem (ILP), and the ILP may be viewed, printed and solved as indicated in the flow chart.

7) Solve ILP

The ILP is solved by means of a software product of ILOG viz CPLEX [13] which is reputed to be state of the art optimization software. The solution to the problem may be viewed in text or as a graphical representation. It is also possible to save and print graphical representations. A table showing the links to be included in the design can be displayed.

8) Save

The network layout can be saved to a file for later use.

9) Network Alteration

Alterations to the network design may be done via the edit options for example disabling a link or node included in the optimal answer.

IV. Illustration of Survivable Network Design Employing 2ECON and 2NCON Concepts

A. Illustrative example

We give now an illustrative example of 2ECON and 2NCON problems based on the model formulation given in section D in II. Figure 1 gives a small network design problem for five nodes.

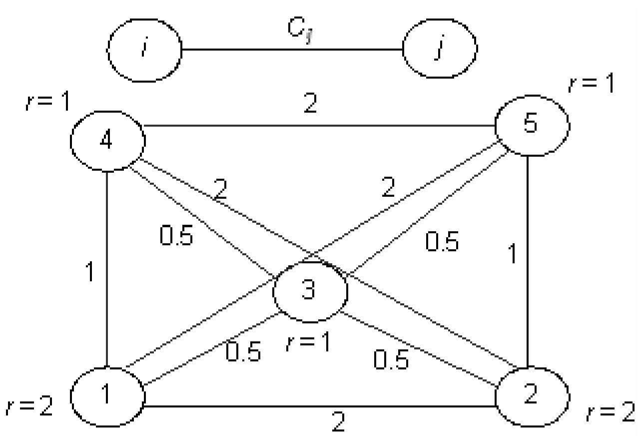

Figure 1 An example of a 5-node network.

In this example we use the above edge costs as the cost for using a particular edge in the network. The *r* denotes the connectivity of the nodes. The cost figures and connectivity of the nodes were chosen to illustrate the 2ECON and 2NCON concepts.

To solve this survivable network problem we begin by constructing edge cut sets and node cut sets. Then constraints are generated from the cut sets, the ILP is constructed and solved with the CPLEX software. A part of the formulation details is given below:

The Network considered is $V = \{1, 2, 3, 4, 5\}$

As a first task to construct constraints (8) in the model all sets W have to be identified. First, we note that the number of such W-sets are:

The number of combinations of set size 1 = 5

The number of combinations of set size 2 = 10

The number of combinations of set size 3 = 10

The number of combinations of set size 4 = 5

The total number of combinations to be considered = 30

Obviously these 30 combinations contain duplicates.

The Output of Set Combinations:

First, taking

$W1 = \{1\}$ $W2 = \{2\}$ $W3 = \{3\}$ $W4 = \{4\}$ $W5 = \{5\}$

We find the edge sets induced by $W1$ to $W5$ as:

$\delta(W1) = \{(1,2)\ (1,3)\ (1,4)\ (1,5)\}$ con $(W1) = 2$

$\delta(W2) = \{(1,2)\ (2,3)\ (2,4)\ (2,5)\}$ con $(W2) = 2$

$\delta(W3) = \{(1,3)\ (2,3)\ (3,4)\ (3,5)\}$ con $(W3) = 1$

$\delta(W4) = \{(1,4)\ (2,4)\ (3,4)\ (4,5)\}$ con $(W4) = 1$

$\delta(W5) = \{(1,5)\ (2,5)\ (3,5)\ (4,5)\}$ con $(W5) = 1$

Similarly for 2-node combinations we have:

$W6 = \{1,2\}$ $W7 = \{1,3\}$ $W8 = \{1,4\}$ $W9 = \{1,5\}$

$W10 = \{2,3\}$ $W11 = \{2,4\}$ $W12 = \{2,5\}$ $W13 = \{3,4\}$

$W14 = \{3,5\}$ $W15 = \{4,5\}$

and

$\delta(W6) = \{(1,3)\ (2,3)\ (1,4)\ (2,4)\ (1,5)\ (2,5)\}$ con $(W6) = 1$

$\delta(W7) = \{(1,2)\ (2,3)\ (1,4)\ (3,4)\ (1,5)\ (3,5)\}$ con $(W7) = 2$

$\delta(W8) = \{(1,2)\ (2,4)\ (1,3)\ (3,4)\ (1,5)\ (4,5)\}$ con $(W8) = 2$

$\delta(W9) = \{(1,2)\ (2,5)\ (1,3)\ (3,5)\ (1,4)\ (4,5)\}$ con $(W9) = 2$

$\delta(W10) = \{(1,2)\ (1,3)\ (2,4)\ (3,4)\ (2,5)\ (3,5)\}$
con $(W10) = 2$

$\delta(W11) = \{(1,2)\ (1,4)\ (2,3)\ (3,4)\ (2,5)\ (4,5)\}$
con $(W11) = 2$

$\delta(W12) = \{(1,2)\ (1,5)\ (2,3)\ (3,5)\ (2,4)\ (4,5)\}$
con $(W12) = 2$

$\delta(W13) = \{(1,3)\ (1,4)\ (2,3)\ (2,4)\ (3,5)\ (4,5)\}$
con $(W13) = 1$

$\delta(W14) = \{(1,3)\ (1,5)\ (2,3)\ (2,5)\ (3,4)\ (4,5)\}$
con $(W14) = 1$

$\delta(W15) = \{(1,4)\ (1,5)\ (2,4)\ (2,5)\ (3,4)\ (3,5)\}$
con $(W15) = 1$

Since the 3-node combinations give the same edge sets as the 2-node combinations, it is not necessary to do them. The same is true for the 4-node combinations as related to the
1-node combinations.

We thus have 15 constraints in (8). The first constraint is for example: C1: y12 + y13 + y14 + y15 >= 2. The full mathematical model is given below as model M. For the 2ECON problem the constraints are given as (M8).

Solving the resulting 2ECON integer linear program gives an optimal solution of 4 for the objective function. Figure 2 gives the solution network of the 2ECON survivable problem. This network is 2ECON survivable but not 2NCON survivable since the two edge-independent paths between nodes 1 and 2 share node 3.

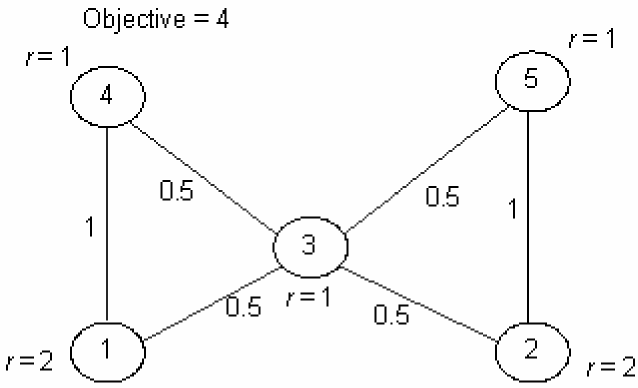

Figure 2 Network solution of 2ECON

To formulate the constraints (9) in the model we note that the information for (8) can be used selectively for each choice of z and W. If we for example take z = 3 and W = {1}, we find $r(W)$ = $r(\{1\})$ = 2 and $r(V\backslash(W \cup z))$ = $r(\{2,4,5\})$ = 2 giving $\delta_{G-z}(W)$ = $\delta_{\{1,2,4,5\}}(\{1\})$ = {(1,2), (1,4), (1,5)} and the constraint N3W6: y12 + y14 + y15 >= 1. The (additional) constraints (9) in the model are given in the full mathematical model below as (M9).

Solving a 2NCON integer linear program problem gives an objective solution of 4 as well. Figure 3 gives the solution network of the 2NCON survivable problem. In this network we have 2-node survivability of nodes 1 and 2 as required.

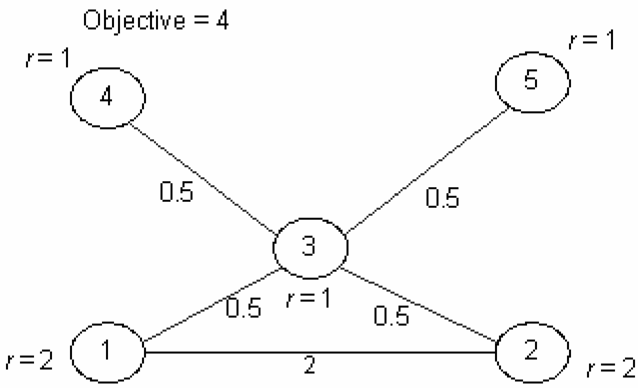

Figure 3 Network solution of 2NCON

The full mathematical model called **model M** is given as an integer linear program:

```
Minimize
Obj: 2y12 + 0.5y13 + y14 + 2y15 + 0.5y23 + 2y24
   + y25 + 0.5y34 + 0.5y35 + 2y45
Subject To
  \\ Edge cut constraints. (M8)
 C1: y12 + y13 + y14 + y15 >= 2
 C2: y12 + y23 + y24 + y25 >= 2
 C3: y13 + y23 + y34 + y35 >= 1
 C4: y14 + y24 + y34 + y45 >= 1
 C5: y15 + y25 + y35 + y45 >= 1
 C6: y13 + y23 + y14 + y24 + y15 + y25 >= 1
 C7: y12 + y23 + y14 + y34 + y15 + y35 >= 2
 C8: y12 + y24 + y13 + y34 + y15 + y45 >= 2
 C9: y12 + y25 + y13 + y35 + y14 + y45 >= 2
 C10: y12 + y13 + y24 + y34 + y25 + y35 >= 2
 C11: y12 + y14 + y23 + y34 + y25 + y45 >= 2
 C12: y12 + y15 + y23 + y35 + y24 + y45 >= 2
 C13: y13 + y14 + y23 + y24 + y35 + y45 >= 1
 C14: y13 + y15 + y23 + y25 + y34 + y45 >= 1
 C15: y14 + y15 + y24 + y25 + y34 + y35 >= 1
   \\ Node cut constraints. (M9)
 N3W1: y12 + y24 + y15 + y45 >= 1
 N3W2: y12 + y25 + y14 + y45 >= 1
 N3W3: y12 + y14 + y25 + y45 >= 1
 N3W4: y12 + y15 + y24 + y45 >= 1
 N3W5: y12 + y24 + y25 >= 1
 N3W6: y12 + y14 + y15 >= 1
 N4W1: y12 + y23 + y15 + y35 >= 1
 N4W2: y12 + y25 + y13 + y35 >= 1
 N4W3: y12 + y13 + y25 + y35 >= 1
 N4W4: y12 + y15 + y23 + y35 >= 1
 N4W5: y12 + y23 + y25 >= 1
 N4W6: y12 + y13 + y15 >= 1
 N5W1: y12 + y23 + y14 + y34 >= 1
 N5W2: y12 + y24 + y13 + y34 >= 1
 N5W3: y12 + y13 + y24 + y34 >= 1
 N5W4: y12 + y14 + y23 + y34 >= 1
 N5W5: y12 + y23 + y24 >= 1
 N5W6: y12 + y13 + y14 >= 1
Binaries  //(M10)
 y12 y13 y14 y15 y23 y24 y25 y34 y35 y45
End
```

B. General solution for 2ECON and 2NCON

Grotschel *et al*. [5] investigate various algorithmic ideas to solve 2ECON and 2NCON problems of the type and size encountered by Bell Communications Research who provided some test data. In this paper we decided to experiment with solutions to the exact ILP problem to ascertain the limitations that a designer can expect from this approach in terms of problem size which we loosely define as the number of nodes and potential edges in the network. This approach is partly motivated by the improvements in computer and software technology over the last few decades.

The idea is then that a network designer could “zoom in” on a portion of a network design problem and try to alleviate the survivability and other problems by using these exact models. This will obviously lead to suboptimal results in the sense that the complete network design problem is not solved but in practical situations this approach may give satisfactory results.

V. Experimental Results

We obtained results for design instances of 6 nodes, 9 nodes, and 12 nodes and up to 15 nodes. Instances with more than 15 nodes could not be solved due to the complexity of the MILP generated. The user has the option of selecting which two nodes he wants to have connectivity of 2 in the proposed design.

The table below gives bounds of time (in seconds) for different instances.

Models	2ECON	2NCON
6 nodes	< 5	< 10
9 nodes	< 10	< 20
12 nodes	< 65	< 120
15 nodes	< 120	…..

The times reported were obtained on an IBM Deskside Server pSeries 630 with AIX version 5.2.

VI. Conclusions

We have found it feasible to solve network topologies for small problems of the types 2ECON and 2NCON using CPLEX, and integrating this into edge-capacitated network flow models is the next challenge.

The results we obtained with the experimental system indicate that such a decision support system can be useful in mesh design networks and notably to fiber optic communication networks with large amounts of traffic carried on each edge compared to traditional band-width-limited technologies in which network survivability may be of less importance. Some flow congestion and relief measures can also be investigated by using the system, if it also has flow dimensioning properties. This is not reported in this paper.

VII. Future Work

The system developed has limitations and relaxing some of the constraints in the current version may contribute to making the system more applicable to large real life problems.

One such constraint is that large network problems cannot presently be solved with exact methods due to the limitation of the current software and computer technology. This aspect certainly would merit further investigation as more advanced computers and software technologies become available.

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