Comparative Analysis of Low Complexity Channel Estimation Techniques for the Pilot-assisted Wireless OFDM Systems

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Abstract—**High-rate wireless orthogonal frequency division multiplexing (OFDM) systems need accurate channel estimation in order to compensate for the distortions caused by propagation through the dispersive channel. This work compares two fundamentally different pilot-assisted channel estimation algorithms, which take into account frequency correlation of the channel: the maximum likelihood (ML) and the linear minimum mean squared error (LMMSE) criteria based. Both performance and computational complexity are analyzed to establish a feasible solution.**

Index Terms—**OFDM, multipath, channel estimation, PSAM.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) technology has become increasingly popular nowadays. IEEE 802.11a/g, ETSI HIPERLAN/2 and IEEE 802.16a standards are the brightest examples of high-rate wireless communication systems based on OFDM.

Contemporary wireless OFDM systems offer high transmission rates due to the use of spectrally efficient Quadrature Amplitude Modulation (QAM), with the dense vector arrangement in the signal constellation. Coherent demodulation of the QAM signals requires explicit knowledge of the channel response, in order to minimize the probability of detection error. Thus, accurate channel estimation is of crucial importance to keep system performance at proper level.

OFDM technology possesses a considerable number of advantages in comparison with traditional singlecarrier systems [4], however some drawbacks are inherent too. Hereafter two of them, namely, increased receiver complexity and sensitivity to time variation of the channel, will be discussed in the context of the channel estimation problem.

Generally, channel estimation methods are split into two major classes: training-based and blind [1][4]. Hereafter we will address only the training-based approach, which relies on the pilot-symbol assisted modulation (PSAM), as blind estimation is seldom used in practical OFDM systems. Moreover, there is another convincing factor from the practical standpoint: transmitting a training sequence effectively helps to achieve receiver synchronization, which is of no less importance for the OFDM systems.

In most application scenarios of the wireless OFDM systems, the propagation channel can be assumed slow fading, which exhibits strong frequency correlation (within one OFDM symbol) and time correlation (across several symbols) properties. Priority must be addressed to the accurate frequency-domain estimation performed on the interval of one OFDM symbol, as it gives better performance for a given complexity, than the approaches exploiting long-term time correlation of the channel. Derivation of such a scheme is presented in [3], where it is referred as the block-oriented linear minimum mean squared error (LMMSE) estimator. The advantage of the LMMSE method is that it belongs to the so-called non-parametric channel estimation techniques, which do not rely on a specific channel model.

Another approach in the OFDM channel estimation deals with parametric channel models. In [2] frequency correlation of the channel is expressed by the finite multipath delay spread. This property allows using deterministic model, parameterized by the channel impulse response, to derive maximum likelihood (ML) estimator. The idea of exploiting ML criterion is that actual estimation can be done in the time domain, where the number of parameters is small, i.e. ML estimator is interpreted as the transformation from frequency domain to time domain and back. In [1] an alternative implementation of the ML estimation algorithm is proposed. Being based on the same idea, it is more suitable for the PSAM systems as it allows overcoming one of the major shortcomings of the previous scheme, i.e. the number of pilot subcarriers to be less than the length of the channel impulse response.

In this paper, we present a comparative analysis of the two fundamentally different low-complexity channel estimation techniques, which can be used in the PSAM-based OFDM systems – low-rank LMMSE and ML. The objective is to answer a number of questions: design limitations, complexity-performance comparison, robustness to changes in channel statistics, etc. The rest of the work is organized as follows. In Section II OFDM system model is described. Section III introduces the channel estimation algorithms. The complexity of these algorithms is analysed in Section IV. Section V contains performance evaluation results obtained by means of simulations. This is followed by conclusions in Section VI.

II. OFDM SYSTEM MODEL

In the given work a single-input-single-output discrete-time baseband OFDM model is considered. It includes transmitter, receiver and equivalent bandlimited channel model (Fig.1). The transmitter and the receiver are assumed to have ideal timing and frequency synchronization.

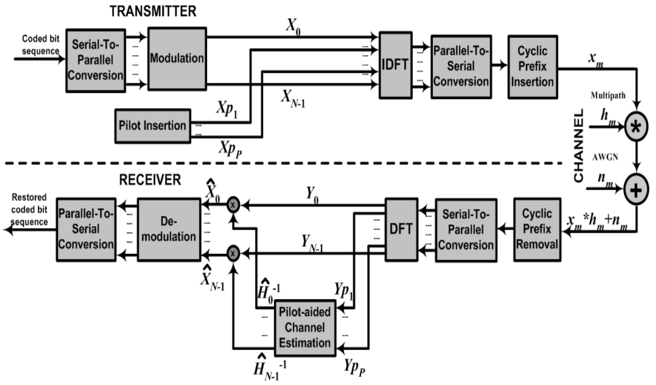

Fig. 1. Baseband OFDM system with PSAM

In the transmitter, serial stream of (coded) data bits is divided into N parallel binary streams, each of which passes through a linear modulation scheme. The OFDM symbol is formed as the result of the inverse discrete Fourier transform (IDFT), applied to N parallel complex-valued modulation symbols X_n, $n = 0,...,N-1$. The resultant waveform is converted to a serial sequence of samples. Before transmission each OFDM symbol is prepended with the cyclic prefix, which is a copy of the last portion of the OFDM symbol. The resultant time-domain waveform of the ith OFDM symbol can be written as

$$x_m(i) = \frac{1}{N}\sum_{k=0}^{N-1} X_k(i) \cdot e^{j\frac{2\pi}{N}k(m-N_{\mathrm{cp}})}, \quad (1)$$

where N_{cp} is the length of the cyclic prefix, $m = 0,...,N+N_{\mathrm{cp}}-1$ is the sample index, and $i = 0,...,K-1$ is OFDM symbol index.

In the considered scenario the channel is assumed to be slowly fading, i.e. the channel response is approximately constant during one OFDM symbol. However, if the channel is time-varying, typically in situations when the transmitter and the receiver are mobile relatively to each other or to the objects on the propagation path, the channel response undergoes strong variations on the interval of one OFDM symbol (fast fading), which lead to the loss of orthogonality between subcarriers and, as a consequence, severe signal distortions due to intercarrier interference (ICI). In [1] it is asserted that a time-varying channel can be well approximated by the time-invariant model during time interval T if

$$T \le 0.01/f_{\mathrm{D}}, \quad (2)$$

where $f_{\mathrm{D}} = f_{\mathrm{c}}v/c$ is the maximum Doppler frequency, f_{c} is the RF carrier frequency, v is the speed of relative movement between the transmitter and the receiver, and c is the speed of light. Criterion (2) is usually satisfied for all fixed or slow-moving high-rate wireless OFDM systems, operating in the band 2-11 GHz, as the duration of the OFDM symbol is much shorter than the coherence time of the radio channel. This allows assuming the channel response approximately time-invariant in the interval of one OFDM symbol. For example, consider IEEE 802.16a system, with the OFDM symbol duration T = 11.46 μs, operating at 11 GHz. If the relative speed of the transmitter-receiver movement is 5 km/h, then T << 0.2 ms, i.e. criterion (2) is true.

Assuming a multipath time-invariant channel, the length of the cyclic prefix N_{cp} can be selected big enough to accommodate finite channel impulse response h_l, $l = 0,...,L$, where the maximum sample-normalized delay spread $L \le N_{\mathrm{cp}}$. Thus, the intersymbol interference (ISI) between consecutive OFDM symbols will be eliminated. Channel impulse response during transmission of the ith OFDM symbol can be written in the form of the coefficients of the discrete-time band-limiting filter with the bandwidth equal to that of the signal:

$$h_m(i) = \begin{cases} h_m(i), & \text{for} \quad m = 0,...,N_{\mathrm{cp}}-1 \\ 0, & \text{for} \quad m = N_{\mathrm{cp}},...,N+N_{\mathrm{cp}}-1 \end{cases}, \quad (3)$$

where $h_m(i)$ are the independently identically distributed zero-mean complex Gaussian variables, with Rayleigh distribution of magnitudes and uniform distribution of phases, and m is referred to as the path index in the equivalent bandlimited multipath channel model. The power delay profile of the channel is assumed to be exponentially decaying, i.e.

$$E\{|h_m(i)|^2\} = C\exp(-m/\tau), \quad (4)$$

where

$$\tau \approx \begin{cases} \tau_{\mathrm{RMS}}, & \text{for} \quad m << L/(2\sqrt{3}) \\ 0.248L\tan[\tau_{\mathrm{RMS}}/(0.265L)], & \text{for} \quad m \le L/(2\sqrt{3}) \end{cases},$$

τ_{RMS} is the sample-normalized root-mean-squared delay spread, and constant C is selected to normalize energy to unity.

The received signal is the sum of the linear convolution of the transmitted one with the channel impulse response and the additive white Gaussian noise (AWGN):

$$r_m(i) = h_m(i) * x_m(i) + n_m(i) = \sum_{l=0}^{L} h_l(i)x_{m-l} + n_m(i), \quad (5)$$

where $m = 0,...,N+N_{\mathrm{cp}}-1$, $i = 0,...,K-1$,

$$x_{m-l} = \begin{cases} x_{m-l}(i), & \mathit{if} \quad m-l \ge 0 \\ x_{m-l+N+N_{\mathrm{cp}}}(i-1), & \mathit{if} \quad m-l < 0 \end{cases}.$$

At the receiver side, after removing cyclic prefix and applying DFT to the ith OFDM symbol we get a vector of received data symbols, expressed in the frequency (parallel) domain as follows:

$$\begin{aligned} Y_n(i) &= \sum_{k=0}^{N-1} r_k(i) \cdot e^{-j\frac{2\pi}{N}k\cdot n} = \\ &= \sum_{k=0}^{N-1}\left(\sum_{l=0}^{L} h_l(i) \cdot x_{k+N_{\mathrm{cp}}-l}(i)\right) \cdot e^{-j\frac{2\pi}{N}k\cdot n} + \sum_{k=0}^{N-1} n_k(i) \cdot e^{-j\frac{2\pi}{N}k\cdot n} = \\ &= H_n(i) \cdot X_n(i) + N_n(i), \quad n = 0,...,N-1 \end{aligned} \quad (6)$$

In the matrix notation:

$$\mathbf{Y}(i) = [Y_0(i) \quad \cdots \quad Y_{N-1}(i)]^T = \mathbf{X}_{[D]}(i) \cdot \mathbf{H}(i) + \mathbf{N}(i), \quad (7)$$

where

$\mathbf{X}_{[D]}(i) = \begin{bmatrix} X_0(i) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & X_{N-1}(i) \end{bmatrix}_{N\times N}$ denotes a diagonal matrix with data symbols,

$\mathbf{H}(i) = [H_0(i) \ \ \cdots \ \ H_{N-1}(i)]^T$ is the DFT of the channel impulse response $h_m(i)$, $m = 0,\dots,N-1$,

$\mathbf{N}(i) = [N_0(i) \ \ \cdots \ \ N_{N-1}(i)]^T$ are the DFT-transformed white noise variables.

Before the parallel set of the received complex symbols Y_n, $n = 0,\dots,N-1$ can be demodulated, i.e. sliced on the reference signal constellation, and converted into the binary decisions, it is necessary to perform equalization, i.e. to correct signal distortions caused by passing through the channel. As OFDM systems work by resolving the frequency domain, a simple block-oriented one-tap equalizer can be used. It divides the received (after DFT) signal by the complex-valued estimate of the channel frequency response (hereafter OFDM symbol index i is omitted for clarity, as the processing is done on a symbol-by-symbol basis, without considering correlation with the neighbouring symbols):

$$\hat{X}_n = Y_n / \hat{H}_n . \qquad (8)$$

Channel estimate $\hat{H}_n$ can be obtained with the help of PSAM. This method relies on the transmission of the known training sequence on a small fraction of subcarriers, usually equally spaced over the whole band (Fig.2), and optionally on varying positions from one OFDM symbol to the next [4].

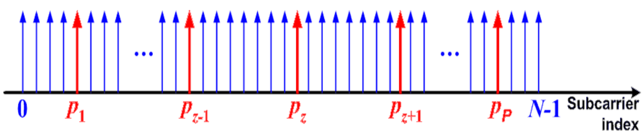

Fig. 2. OFDM spectrum with equally spaced pilot subcarriers

III. Channel Estimation

A. Linear Interpolation of the LS Estimates

In the simplest case the channel estimates are found by straightforward multiplying the received pilot symbols by the inverse of the reference pilot symbol values – the so-called least squares (LS) estimator, which can be written as

$$\hat{\mathbf{H}}_{\mathbf{P}}^{\mathrm{LS}} = \mathbf{X}_{\mathbf{P}_{[D]}}^{-1}\mathbf{Y}_{\mathbf{P}} = \left[X_{p_1}^{-1}Y_{p_1} \ \ \cdots \ \ X_{p_P}^{-1}Y_{p_P}\right]^T , \qquad (9)$$

where $\{p_1,\dots,p_P\}$ denotes the set of subcarriers, which are used to carry pilot symbols as illustrated in Fig.2.

After that, the channel estimates $\hat{H}_{p_z}^{\mathrm{LS}}, z = 1,\dots,P$, obtained at the pilot positions, are interpolated over the whole band. Linear interpolation represents an example of a solution with the least possible computational complexity, when only one multiplication by a real factor is needed to compute the channel estimate for each data subcarrier:

$$\hat{H}_{p_z+k}^{\mathrm{LS}} = \hat{H}_{p_z}^{\mathrm{LS}} + \frac{k}{p_{z+1} - p_z}\left(\hat{H}_{p_{z+1}}^{\mathrm{LS}} - \hat{H}_{p_z}^{\mathrm{LS}}\right), \qquad (10)$$

where $k = 1,\dots,p_{z+1} - p_z - 1$, and $p_{z+1} - p_z = \text{const} \ \ \forall\, z = 1,\dots,P$ for the case of the equally-spaced pilot subcarriers. Pilot symbols at the positions p_1 and p_P are treated as a neighbour pilot pair for interpolation of the subcarriers on the spectrum edges.

It is reasonable that the main disadvantage of the linear-interpolated LS estimator is its poor performance, as it takes into account neither statistical, nor structural properties of the channel. We include the LS scenario for the purpose of illustrating the performance bound for the smallest complexity. In practical systems more accurate channel estimation algorithms should be used.

B. LMMSE Estimator

The linear minimum mean squared error (LMMSE) estimator, concerned in this paper, is designed to work in the frequency-domain only (one-dimensional). It represents a parallel-type linear filter (FIR filter analogue in the frequency domain), which provides minimization of the mean squared error appearing in the result of the LS estimation during one OFDM symbol. For the case, when all subcarriers are used as pilots, this algorithm is given by [3]:

$$\hat{\mathbf{H}}^{\mathrm{MMSE}} = \mathbf{R}\left[\mathbf{R} + \sigma_{\mathrm{noise}}^2\left(\mathbf{X}_{[D]}^H\mathbf{X}_{[D]}\right)^{-1}\right]^{-1}\hat{\mathbf{H}}^{\mathrm{LS}} , \qquad (11)$$

where $\mathbf{R} = E\{\mathbf{H}\mathbf{H}^H\}$ is the $N\times N$ channel autocorrelation matrix, $\sigma_{\mathrm{noise}}^2$ is the variance of the additive Gaussian noise, $(.)^H$ denotes Hermitian (conjugated) transpose, $\hat{\mathbf{H}}^{\mathrm{LS}} = \left[\hat{H}_0^{\mathrm{LS}} \ \ \cdots \ \ \hat{H}_{N-1}^{\mathrm{LS}}\right]^T$ is the LS estimate of the channel transfer function given by (9), but for the all-pilot case.

For the PSAM-based OFDM system, (11) can be modified to specify a pilot approximator, which uses only P LS estimates as input values:

$$\hat{\mathbf{H}}_{\mathbf{P}}^{\mathrm{MMSE}} = \mathbf{Q}\hat{\mathbf{H}}_{\mathbf{P}}^{\mathrm{LS}} = \mathbf{R}_{\mathbf{P}}\left[\mathbf{R}_{\mathbf{PP}} + SNR^{-1}\mathbf{I}\right]^{-1}\hat{\mathbf{H}}_{\mathbf{P}}^{\mathrm{LS}} , \qquad (12)$$

where the $N\times P$ matrix $\mathbf{R}_{\mathbf{P}} = E\{\mathbf{H}\mathbf{H}_{\mathbf{P}}^H\}$ is obtained from decomposition of $\mathbf{R}$ involving permutation of columns, i.e. $\mathbf{R}_{\mathbf{P}}^{<z-1>} = \mathbf{R}^{<p_z>}$, $z = 1,\dots,P$, with $(.)^{<.>}$ denoting a column of the matrix with the given index; $\mathbf{R}_{\mathbf{PP}} = E\{\mathbf{H}_{\mathbf{P}}\mathbf{H}_{\mathbf{P}}^H\}$ is the $P\times P$ matrix, obtained from decomposition of $\mathbf{R}_{\mathbf{P}}$ involving permutation of rows, i.e. $\mathbf{R}_{\mathbf{PP}}{}^{<z-1>} = (\mathbf{R}_{\mathbf{P}}^H)^{<p_z>}$, $z = 1,\dots,P$ (note that $\mathbf{R}$ is Hermitian); and $\mathbf{I}$ is the $P\times P$-size identity matrix. The term SNR in (12) is the ratio of the signal power $E\{|X_{p_z}|^2\}$ to the noise power $\sigma_{\mathrm{noise}}^2$. Equation (12) corresponds to the case when pilot symbols transmitted on different subcarriers have equal power. If, however, it is not so, then the matrix $SNR^{-1}\mathbf{I}$ in (12) has to be replaced by $\sigma_{\mathrm{noise}}^2\left(\mathbf{X}_{\mathbf{P}_{[D]}}^H\mathbf{X}_{\mathbf{P}_{[D]}}\right)^{-1} \approx \dfrac{E\{|X_{p_z}|^2\}E\{1/|X_{p_z}|^2\}}{SNR}\mathbf{I}$.

The LMMSE estimator (12) uses *a priori* knowledge of the signal-to-noise ratio and the channel autocorrelation matrix $\mathbf{R}$, and is optimal when the statistical properties of the channel are known. Here the SNR value can be predefined: higher SNR ratios are preferable to obtain more accurate estimates. The robust estimator design necessitates account for the worst correlation of the multipath channel, namely when the channel power-delay profile represents the uniform function. Under such an assumption the elements of the channel correlation matrix are expressed as follows:

$$R_{k,l} = \frac{1-\exp\left(-j2\pi\frac{k-l}{N}N_{\mathrm{cp}}\right)}{j2\pi\frac{k-l}{N}N_{\mathrm{cp}}}, \quad k,l = 0,...,N-1\,. \tag{13}$$

Straightforward product with the weighting matrix $\mathbf{Q}$ in the expression (12) involves *NP* complex multiplications that represent a considerable computational load if *P* is large. In order to reduce it, [3] proposes to apply an optimal rank reduction for the matrix $\mathbf{Q}$, which is based on the singular value decomposition (SVD). This lets $\mathbf{Q}$ to be written as a product

$$\mathbf{Q} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H, \tag{14}$$

where $\mathbf{\Lambda}$ is the $N\times P$-size diagonal matrix containing the singular values $\lambda_0 \ge \lambda_1 \ge ... \ge \lambda_{P-1}$, $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices of the sizes $N\times P$ and $P\times P$ correspondingly, whose columns are the singular vectors. Interpreting $\mathbf{\Lambda}$ as a descending set of variances (powers) of the linear transform coefficients, one can exclude all but the *r* largest singular values $\lambda_0 \ge ... \ge \lambda_{r-1}$, i.e. $\mathbf{\Lambda}$ can be decomposed as $\mathbf{\Lambda} = \begin{vmatrix}\mathbf{\Lambda_r} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{vmatrix}_{N\times P}$, where $\mathbf{\Lambda_r}$ is the $r\times r$-size diagonal matrix with elements $\lambda_0 \ge ... \ge \lambda_{r-1}$. Thus, the rank of the matrix $\mathbf{Q}$ is reduced from P to r ($r \le P$). Simulation experiments (Section V) show, that setting $r = L + \Delta = N_{\mathrm{cp}} + \Delta$, where $\Delta = 5$ for target SNR ≤ 40 dB, ensures accurate low-rank approximation without the error floor effect due to ignoring coefficients with smaller magnitudes ($\lambda_r \ge ... \ge \lambda_{P-1}$), and further increase of *r* does not give noticeable performance improvement as the remaining coefficient magnitudes are very close to zero. There is, however, a restriction that the number of pilot subcarriers *P* must be no less than $r = N_{\mathrm{cp}} + \Delta$ for an accurate estimator construction.

Notation (12) can be rewritten [6] as a combination of the orthogonal singular vectors $\mathbf{u}_i = \mathbf{U}^{<i>}$ and $\mathbf{v}_i = \mathbf{V}^{<i>}$, then

$$\begin{aligned}\hat{\mathbf{H}}_{\mathbf{P}}^{\mathrm{MMSE}} &= \mathbf{U}\begin{vmatrix}\mathbf{\Lambda_r} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{vmatrix}\mathbf{V}^H\hat{\mathbf{H}}_{\mathbf{P}}^{\mathrm{LS}} = \\ &= \left(\sum_{i=0}^{r-1}\lambda_i\mathbf{u}_i\mathbf{v}_i^H\right)\hat{\mathbf{H}}_{\mathbf{P}}^{\mathrm{LS}} = \sum_{i=0}^{r-1}(\lambda_i\mathbf{u}_i)\left\langle\mathbf{v}_i, \hat{\mathbf{H}}_{\mathbf{P}}^{\mathrm{LS}}\right\rangle\end{aligned}, \tag{15}$$

where $\lambda_i\mathbf{u}_i$ is the scalar-vector product, and $\left\langle\mathbf{v}_i, \hat{\mathbf{H}}_{\mathbf{P}}^{\mathrm{LS}}\right\rangle$ is the inner Euclidean product of the two vectors. Note that the vectors $\lambda_i\mathbf{u}_i$ and $\mathbf{v}_i$, where $i = 0,..,r-1$, are pre-computed at the design stage from SVD of $\mathbf{Q}$, based on the preset target SNR and the channel correlation matrix, and stay fixed during the estimator's operation.

The functional diagram of the low-rank pilot-assisted LMMSE channel estimator (15) is depicted in Fig.3. One can see that the LS estimates of the pilots are projected onto a (smaller) subspace spanned by the vectors $\mathbf{v}_i$, where estimation is performed. The final channel estimates are then found by linear combination of the basis vectors $\mathbf{u}_i$.

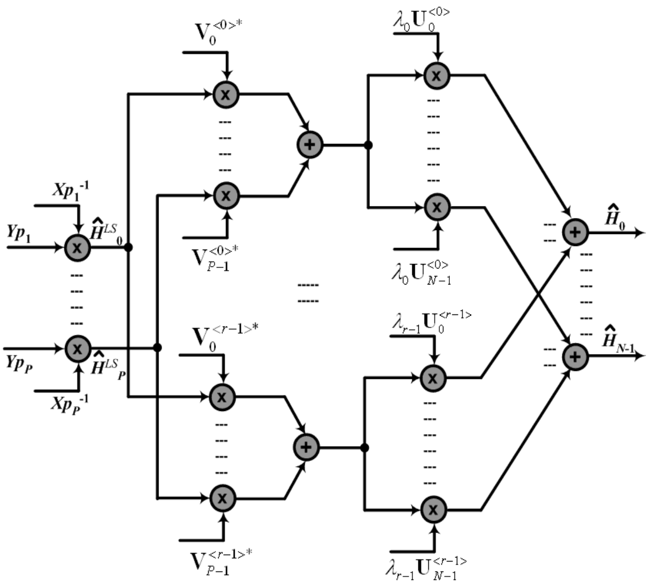

Fig. 3. Block diagram of the reduced-rank PSAM-driven LMMSE channel estimator

C. ML Estimator

An alternative in the channel estimator choice is the maximum likelihood (ML) criterion based approach [1]. The main idea is to obtain an estimate of the channel impulse response $\mathbf{h}$ and data symbols $\mathbf{X}$ vector that jointly minimize the Euclidean distance function

$$d(\mathbf{h},\mathbf{X}) = \sum_{n=0}^{N-1}\left|Y_n - \sum_{l=0}^{N_{\mathrm{cp}}-1}h_l e^{-j\frac{2\pi}{N}ln}X_n\right|^2. \tag{16}$$

The ML estimation algorithm exploits the deterministic property of the limitedness of the channel impulse response, when the largest sample-normalized delay spread *L* is assumed to be less than the length of the cyclic prefix N_{cp}, i.e. $L \le N_{\mathrm{cp}} - 1$. This structural feature is closely linked with the frequency correlation of the channel, and, if known precisely, allows to construct an optimal estimator without any other knowledge of the channel [2].

Performing minimization of (16) provided that $\mathbf{X}$ is known (as in the case when all subcarriers are used as pilots), leads to the following expression of the algorithm's output, written in a matrix notation:

$$\hat{\mathbf{H}}^{\mathrm{ML}} = \sqrt{\frac{N}{N_{\mathrm{cp}}}}\mathbf{F}'\mathbf{\Phi}^H\left(\mathbf{\Phi}\mathbf{F}^H\mathbf{X}_{[D]}^H\mathbf{X}\right)_{[D]}^{-1}\mathbf{\Phi}\mathbf{F}^H\mathbf{X}_{[D]}^H\mathbf{Y}, \tag{17}$$

where $\mathbf{F}$ is the orthonormal Fourier matrix with elements $F_{k,l} = \frac{1}{\sqrt{N}}e^{-j\frac{2\pi}{N}kl}$, $k,l = 0,...,N-1$; $\mathbf{F}'$ is the Fourier matrix, which is truncated to the size $N\times N_{\mathrm{cp}}$, i.e. $\mathbf{F} = \left|\mathbf{F}' \;\; \cdots\right|$; $\mathbf{\Phi} = \left|\mathbf{F_{cp}} \;\; \mathbf{0}\right|$ is the catenation of the N_{cp}-size orthonormal Fourier matrix $\mathbf{F_{cp}}$ with elements $F_{\mathrm{cp}\,\mu,\nu} = \frac{1}{\sqrt{N_{\mathrm{cp}}}}e^{-j\frac{2\pi}{N_{\mathrm{cp}}}\mu\nu}$, $\mu,\nu = 0,...,N_{\mathrm{cp}}-1$, and the $N_{\mathrm{cp}}\times(N-N_{\mathrm{cp}})$-size zero matrix; subscript $(.)_{[D]}$ denotes conversion of a vector into the corresponding diagonal matrix.

A flow chart visualizing the described ML algorithm is shown in Fig.4. The overall algorithm procedure represents a repeated translation from the frequency domain to the time-domain and back using two Discrete Fourier Transform (DFT/IDFT) sets of different sizes. The actual estimation is performed in the time domain, where the number of parameters ($\hat{h}_0,...,\hat{h}_{N_{\text{cp}}-1}$) is substantially smaller than in the frequency domain ($\hat{H}_0,...,\hat{H}_{N-1}$). This comes to a product in (17) with the weighting matrix $\mathbf{\Phi}^H \left(\mathbf{\Phi F}^H \mathbf{X}_{[D]}^H \mathbf{X}\right)_{[D]}^{-1} \mathbf{\Phi}$, where the inverse term $\left(\mathbf{\Phi F}^H \mathbf{X}_{[D]}^H \mathbf{X}\right)_{[D]}^{-1}$ has a small dimension and can be pre-calculated and stored as a set of N_{cp} weight coefficients.

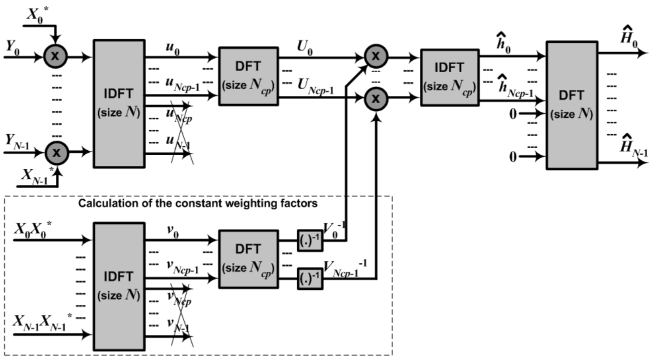

Fig. 4. Block diagram of the ML estimator

Estimator (17) is easily adapted to the PSAM case. If only P subcarriers are used for transmission of the training information, then solely pilot symbols given by the vectors $\mathbf{Y_P}$ (received) and $\mathbf{X_P}$ (reference) are fed to the input of the estimator, whereas symbols at the positions of data subcarriers are replaced by zeros, i.e. $X_n = \begin{cases} X_{p_z}, & \text{for } n = p_z \\ 0, & \text{for } n \neq p_z \end{cases}$, where $z = 1,...,P$, and $n = 0,...,N-1$.

IV. Algorithm Complexity

In this section the above channel estimation algorithms are analyzed from the standpoint of computational complexity, which is traditionally expressed as a number of complex multiplications (*CMs*) required to obtain an estimate of the channel transfer function on the interval of one OFDM symbol. In general, for the PSAM-based OFDM systems this number depends on the quantity of pilot subcarriers in the spectrum, and the maximum delay spread of the channel (cyclic prefix length).

A. Low-rank LMMSE Estimator

The form of the estimator given in (15) involves P CMs to compute the inner product $\left\langle \mathbf{v}_i, \hat{\mathbf{H}}_{\mathbf{P}}^{\text{LS}} \right\rangle$ for each vector $\mathbf{v}_i$. The resultant value is multiplied with the corresponding vector $\lambda_i \mathbf{u}_i$, representing $(N-P)$ CMs needed to obtain estimates for the $(N-P)$ subcarriers transmitting data. As the final estimates are computed by a sum of r vectors, the total number of CMs per OFDM symbol becomes $r(N-P+P)+P = rN+P$. Here the last summable term P is the number of LS estimates on the pilot subcarriers. Assuming that the rank of the LMMSE approximator is equal to $r = L + \Delta = N_{\text{cp}} + \Delta$ where $\Delta = 5$, the complexity is expressed as $(N_{\text{cp}}+5)N+P$ CMs per symbol. However, in our analysis we express the CM number for the low-rank LMMSE estimator being equal to $(N_{\text{cp}}+4)N+P$ for the proper comparison with the ML estimator, which, as mentioned previously (Section III), requires the maximum delay spread of the channel to be no more than $L = N_{\text{cp}} - 1$.

B. ML Estimator

The time-frequency interpretation based on the pairs of the Fast Fourier Transforms significantly diminishes complexity of the ML estimation algorithm. The conventional radix-4 implementation of the FFT/IFFT, which forms the base of most contemporary OFDM systems and requires approximately $3N/4(\log_4 N - 1)$ CMs, can be efficiently used for the translation of the N input values $Y_n X_n^*$ to the time domain and converting back to the frequency domain at the final processing stage (Fig.4). Furthermore, as reported in [2] a comb-optimized inverse FFT engine can be used for the PSAM spectrum with equally spaced pilot subcarriers, which needs only about $N/4 + \log_2 N_{\text{cp}} - N_{\text{cp}}/2$ CMs and represents a large gain for the big number of subcarriers N. For the DFT/IDFT of the N_{cp} size the radix-2 FFT implementation has to be used as in general the length of the channel response may not be a power of 4. It requires about $N_{\text{cp}}/2(\log_2 N_{\text{cp}} - 1)$ CMs. The remaining computational operations of the described ML algorithm include P CMs at the estimator's input and N_{cp} CMs with the weighting factors. The approximate total CM amount of the ML estimator is calculated as $1.5N(\log_4 N - 1) + N_{\text{cp}} \log_2 N_{\text{cp}} + P$ for the case of unmodified radix-4 and radix-2 FFTs, and as $N(0.75\log_4 N - 0.5) + (N_{\text{cp}}+1)\log_2 N_{\text{cp}} - N_{\text{cp}}/2 + P$ for the IFFT implementation, optimized for a comb spectrum.

The graphs illustrating complexity of the estimators are shown in Fig.5. The last term P in the total CM number expressions for all the algorithms is omitted. Indeed, its contribution is negligible as the number of pilots is several orders of the magnitude smaller than the total number of CMs required by an estimator. Therefore varying the number of pilot subcarriers does not substantially affect the computational complexity of the considered algorithms.

One can see that the FFT-based ML estimation has lower complexity than the low-rank LMMSE scheme. The increase of the length of the cyclic prefix, required to accommodate ISI due to more extensive channel dispersion, leads to even greater (up to an order of magnitude) complexity difference between the ML and the LMMSE estimator. It should be noted that for a big number of subcarriers in the OFDM spectrum, the quantity of CMs required by the ML algorithm is almost independent of N_{cp}, unlike the LMMSE case, in which it grows proportionally to the length of the channel impulse response.

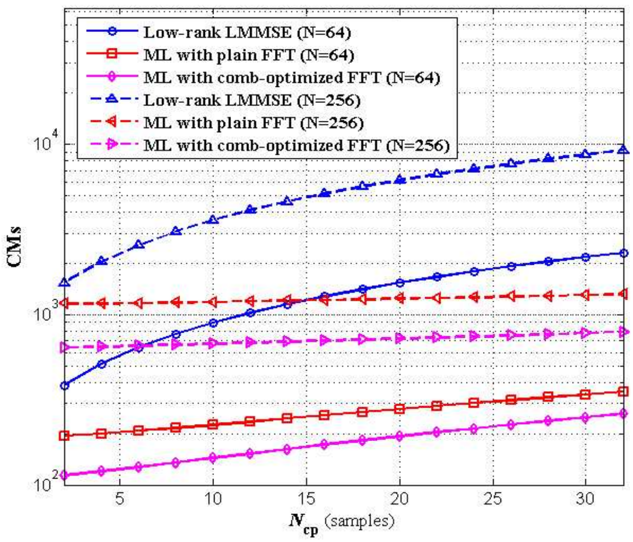

Fig. 5. Computational complexity of the channel estimation algorithms

V. PERFORMANCE ANALYSIS

In the following section we discuss the setup and performance evaluation of the simulated OFDM system, with different channel estimators. To ensure appropriate conditions for the experimental algorithm comparison, it is important to consider effects of the designed estimator and channel mismatch first.

A. Design Considerations

As mentioned previously, the robust implementation of the ML algorithm (17) requires the maximum sample-normalized delay spread of the multipath channel to be equal to $L \leq N_{\rm cp}-1$. Normally this condition is satisfied by selection of the proper cyclic prefix length during the design of the OFDM system. If, however, in some cases the length of the cyclic prefix appears to be insufficient and certain ISI amount affects received OFDM symbols, then it is worthwhile to set the size of the $N_{\rm cp}$-sized FFTs in the ML estimator scheme (Fig.4) to the next power of 2. Simulations show that setting their size to be smaller than L (provided that $L \leq N_{\rm cp}$, in order to exclude ISI) results in irreducible error floor.

Unlike the ML estimator with only one unknown parameter L, the LMMSE estimator (12) has two unknown quantities, namely SNR and channel autocorrelation matrix $\mathbf{R}$. Typically it is sufficient for the robust estimator to set SNR to be equal or higher than the target SNR value, inherent to the case of the largest received signal power. Though it is preferable to have SNR tracking at the receiver (like in case of OFDM systems with adaptive bit loading), which would allow adjusting the weighting matrix in (12) according to the upper SNR threshold for the current transmission mode. Note that the choice of the SNR value, which is smaller than the actual signal-to-noise ratio, results in irreducible error floor [3].

Experiments show that the choice of the autocorrelation matrix $\mathbf{R}$ corresponding to the uniform power-delay profile ensures good LMMSE estimator performance for any channel dispersion profile. However, the major problem here is the accurate estimation of the maximum delay spread (L) of the channel as performance deteriorates if there is a significant difference between L and $N_{\rm cp}$, for which $\mathbf{R}$ (13) is calculated. The latter statement represents the link between statistical and structural (deterministic) channel properties.

Another problem for the LMMSE estimator used in the PSAM systems is the lower bound for the number of subcarriers, which can be used as pilots. This bound is equal to the minimum rank $r = N_{\rm cp} + \Delta$ required for the accurate approximation. The number of pilot subcarriers P can still be decreased, but it requires to reduce the approximating rank of the matrix $\mathbf{Q}$ below $r = N_{\rm cp} + \Delta$ that is achieved by decreasing Δ in (15) to $\Delta = 1$ [3], followed, if necessary, by setting a smaller $N_{\rm cp} = N_{\rm cp}' = P-1$ value in (13). Note that the actual cyclic prefix length in the system remains unchanged. Simulations show that if the RMS delay spread of the channel is small in respect to $N_{\rm cp}'$ (ideally $\tau_{\rm RMS} << N_{\rm cp}'$), performance degradation due to design mismatch is still tolerable.

B. Performance over the Slow Fading Multipath Channel

The simulation scenario consists of a PSAM-based OFDM system with 256 subcarriers and equal spacing of pilots (16 and 32 pilot cases are considered). Subcarriers transmitting uncoded data are modulated by QAM-16. The power of pilot subcarriers is constant. The length of the cyclic prefix is 8 samples. We choose the 8-path bandlimited channel model ($L = 7$) with the exponentially decaying power-delay profile and the sample-normalized delay spread equal to $\tau_{\rm RMS} = 2$. The largest Doppler frequency is modelled according to the equation $f_{\rm D} = 0.001/T$, where T is the OFDM symbol duration (including cyclic prefix). The target E_b/N_0 ratio for the LMMSE estimator was set to 30 dB, and the design was made for the channel with the uniform power-delay profile with the maximum delay spread equal to $N_{\rm cp} = 8$. The rank of the LMMSE estimator was reduced to $r = N_{\rm cp} + 5 = 13$.

Simulation results are shown in Fig.6. Note that here we use the energy-per-bit-to-noise-ratio metric, which is common for performance analysis of the digital communication systems, and is linked to SNR of the PSAM-driven OFDM transmissions by equation [5]

$$\frac{E_{\rm b}}{N_0} = SNR\frac{B}{R} = SNR\frac{BT}{k(N-P)} = SNR\frac{N+N_{\rm cp}}{k(N-P)}, \qquad (18)$$

where $B = 1/T_{\rm s}$ is the bandwidth of the system, $T = (N + N_{\rm cp})T_{\rm s}$ is the OFDM symbol period, and k is the number of bits carried by one modulation symbol ($k = 4$ for QAM-16).

The ML estimator exhibits better performance than low-rank LMMSE for both 16 and 32 pilot subcarriers. The difference with the case when channel response is ideally known constitutes 1.5 dB on average. Note that the number of pilots does not substantially impact performance of the ML estimator. This feature plus lower complexity of the ML algorithm makes it attractive for the cases when OFDM system, driven by a small number of pilots, has to operate over dispersive channels with large maximum delay spreads. For the low-rank LMMSE algorithm, bigger amount of pilot subcarriers ($P = 32$) improves accuracy at higher E_b / N_0 values, so that BER converges with the ML solution.

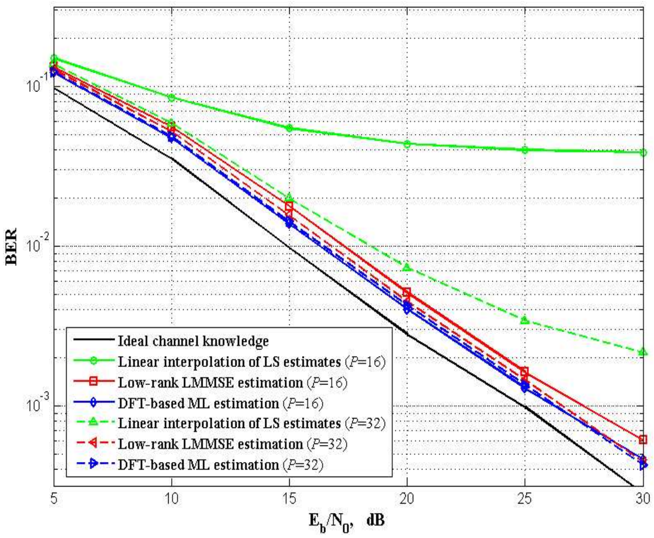

Fig. 6. Performance of different channel estimation algorithms

C. *Effect of time variation of the channel*

In the previous simulation scenario channel was modelled as slow fading with the Doppler spread $f_D = 0.001/T$, i.e. by order of a magnitude smaller than required by (2) that makes its properties almost the same as of the time-invariant channel. An essential question is how channel estimators behave if time variation of the channel becomes stronger. To answer it we simulate OFDM system with the same setup over the same channel but with $f_D = 0.01/T$. Performance evaluation results are presented in Fig.7.

Note that ICI, resultant from the loss of orthogonality of subcarriers due to their Doppler shifts, leads to irreducible error floor for all the considered channel estimation schemes. It is manifested by greater performance difference in comparison with the equalizer using ideal channel knowledge (e.g., approximately 2.7 dB at BER = 10^{-3} for the ML estimator). Subject to ICI distortion, estimates of the channel transfer function on the pilot subcarriers become more erroneous, so that performance difference of the low-rank LMMSE estimator with larger and smaller number of pilots diminishes.

It is reported that computing ML estimates iteratively can increase accuracy [1], however it will not eliminate the error floor due to ICI and, as experiments show, may lead to even worse results if erroneous estimates (for example, affected by noise) obtained from the first iteration are applied to the estimator input for the second iteration.

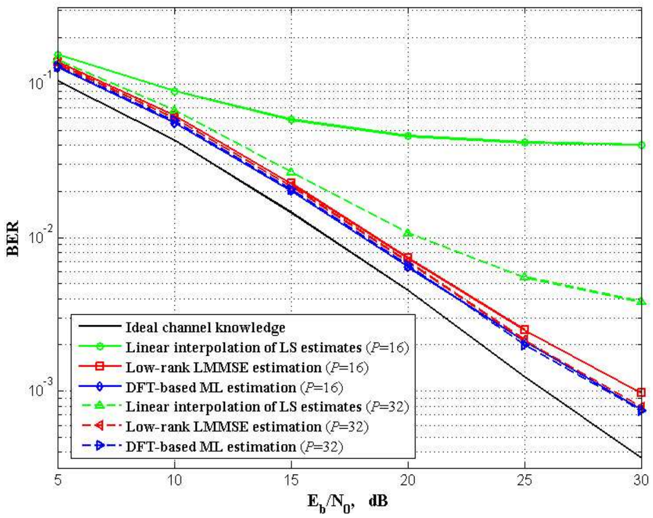

Fig. 7. Performance over the channel with faster time variation

VII. CONCLUSIONS

In this work two fundamentally different OFDM channel estimation algorithms have been compared. One of them (LMMSE) takes into account statistical properties of the channel, while the second (ML) handles deterministic channel features. Both complexity and the number of parameters to be known by the ML algorithm are much less than in case of the low-rank LMMSE estimator. The ML estimator outperforms the low-rank LMMSE, and represents a good solution for OFDM systems operating in the slow fading conditions. In the channel with more intensive time variations additional ICI compensation methods should be used to keep performance at the acceptable level.

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