Low Complexity Adaptive Frequency-Domain Equalizer for Quasi-Orthogonal Space-Time Block Code over Frequency-Selective Channels

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***Abstract*—This paper investigate the performance of an adaptive frequency-domain equalizer for quasi-orthogonal space-time block coded (QSTBC) over frequency selective fading channels. The QSTBC takes the advantage of the intrinsic orthogonal space-time block code (OSTBC) within it to reduce the complexity of the recursive least square (RLS) to that of least mean square (LMS) without compromising performance. The receiver with perfect channel state information is compare to the adaptive receiver under investigation. Also the effects of some of the parameters of the receiver are presented in the simulations. The simulation results for the OSTBC adaptive receiver are used as benchmark to compare with receiver under investigation.**

***Index Terms*— Adaptive receiver, Equalization, Frequency selective channels, MIMO system, Multi-antenna systems, Quasi-orthogonal STBC, SC-FED, Space-time block code.**

I. Introduction

WIRELESS radio channels are ever so attractive even though they are the most unproductive, erroneous and more prone to interference. All these factors contribute in the statistical nature of wireless channels which greatly reduces capacity. Multiple input multiple output (MIMO) systems were introduced to try and increase capacity and reliability of wireless channels.

In [1] Alamouti proposed a simple space-time block coded (STBC) transmission scheme for two transmit antennas. This scheme obtained full diversity in narrow band flat fading channel at full rate however it was unable to obtain multi-path diversity when applied to frequency selective channels. This scheme was later generalized in [2] by noticing the orthogonal property inherent in the scheme. The STBCs of more than two antennas suffers from reduced rate and diversity in some cases – especially for complex constellation. Thus leaving the STBCs with two main problems; first reduced rate for more than two antennas and second loss of multi-path diversity on frequency selective channels.

The first problem was addressed in [3] by Jafarkhani who proposed a full rate quasi-orthogonal transmission scheme for four antennas at an expanse of some diversity. This scheme utilized Alamouti's orthogonal two antenna scheme which had an effect of relaxing orthogonality requirements. Though it is not the main focus of this paper, [4] was able to obtain maximum diversity through constellation rotation. In [5] the second problem of STBCs is addressed by introducing a time-reversed (TR) block transmission of data instead on individual symbols. Enhanced Data rates for Global Evolution (EDGE) is the third generation time division multiple access (TDMA) cellular standard that present a challenging equalization problem. This is due to due to the use of 8-PSK modulation and general non minimum-phase characteristics of the typical urban (TU) channel and addition inter-symbol interference (ISI) due to the Gaussian minimum shift keying [5]. The single-carrier frequency-domain equalization (SC-FED) schemes with cyclic prefix [6] and zero post-fix [7] were developed to address problems of an EDGE channel. The benchmark performance of these schemes of STBCs in frequency selective channels was later generalized in [8].

In [9] Al-Dhahir et al. observed that by properly modifying the quasi-orthogonal transmission scheme proposed in [3], and applying SC-FED scheme with CP insertion at transmitter and through careful manipulation with linear operations of the received signal it is possible to obtain two Alamouti-like (quadtronic) structure. Exploiting this structure it is possible to apply a suitable equalizer to obtain the original data. Mainly all of the above mentioned schemes require channel state information (CSI) and there mainly three ways in which the receiver obtain this information. First is blind approach where the receiver blindly estimates the channel by itself. Though this approach gives the best performance it usually requires complex computation at the receiver. The second is the semi-blind approach where transmit some of the state information with the data so as to reduce complexity. The third is the non-blind (training) CSI estimation where the receiver depends entirely on the transmitter. This approach often gives the worse performance due to system overhead.

In this paper a semi-blind approach to CSI estimation is employed through the use of the RLS algorithm. This algorithm is applied to the situation in [9] – SC-FED for

Manuscript received April 19, 2007. This work was supported in part by the Telkom South Africa Ltd and Alcatel Altech Telecoms as part of the Centre of Excellence program.

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QSTBC over frequency selective channels – to intentionally exploit the inherent quadtronic structure that is obtained after manipulation of the observed signal at the receiver. This property gives RLS performance at LMS complexity. System performance with perfect CSI is obtained. The algorithm's performance results obtained in [10] and that of the perfect CSI [5] of the same situation are presented as benchmark in this paper.

The rest of the paper is organized as follows. In Section II the SC-FED for QSTBC over frequency selective channels is presented along with theory and implementation assumptions made. In Section III the receiver is presented in an MMSE sense while Section IV covers the adaptive receiver for training and tracking mode as described in [10]. Section V then provides simulation results and conclusions are drawn in Section VI.

Notation: Upper case letters represents matrices except when they are italics in which case they represent indices, vector or matrix sizes. Lower case letters represent vectors while italicized lower letters represents sequences. Also $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^\dagger$ represent conjugate, transpose and conjugate transpose, respectively while $[\cdot]_p$ denotes the $(p+1)$th entry of a vector; I_J denotes an identity matrix of size $J\times J$. F_J stands for an $J\times J$ *Fast Fourier Transform* (FFT) matrix.

II. Transmission Scheme

A. QSTBC Scheme

As specified in [3] we assume an ST block coded transmission scheme using four transmit antennas and a single receive antenna. The channel is assumed to be frequency selective but constant over each block transmitted. The QSTBC transmission scheme proposed in [3] is as follow;

$$A=\begin{bmatrix} A_{12} & A_{34} \\ -A_{34}^* & A_{12}^* \end{bmatrix}=\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{bmatrix}, \tag{1}$$

where $A_{12}=\begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}$ and $A_{34}=\begin{bmatrix} x_3 & x_4 \\ -x_4^* & x_3^* \end{bmatrix}$.

Al-Dhahir et al. were able to slightly modify the QSTBC scheme of (1) such that it still remained qusi-orthogonal but much easier to decouple the observed signal at the receiver in such a manor that the manipulated data is though it was sent by a two transmit antenna encoded with the Alamouti scheme of [1] but observed after four transmit intervals. This scheme is may be represented as follow,

$$A=\begin{bmatrix} A_{12} & A_{34} \\ A_{12} & -A_{34} \end{bmatrix}=\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ x_1 & x_2 & -x_3 & -x_4 \\ -x_2^* & x_1^* & x_4^* & -x_3^* \end{bmatrix}. \tag{2}$$

B. Permutation Matrix

A permutation matrix $\mathbf{P}$ is usually drawn from a set of matrices $\{P_J^{(n)}\}_{n=0}^{J-1}$ with J denoting the dimensionality of $J\times J$. Each $P_J^{(n)}$ performs a reverse cyclic shift (that depends on n) when applied to $J\times 1$ vector. Consider a vector $\mathbf{a}\equiv[a(0),a(1),...,a(J-1)]^T$. Specifically $(p+1)$th entry of $P_J^{(n)}\mathbf{a}$ is $[P_J^{(n)}\mathbf{a}]_p=a((J-p+n-1)\bmod J)$ there are two special cases that are usually of interest namely they are $P_J^{(0)}$ and $P_J^{(1)}$ cases. When both are applied to vector $\mathbf{a}$ the following is obtained,

$$P_J^{(0)}\mathbf{a}=[a(J-1),a(J-2),...,a(0)]^T$$

$$\begin{aligned} P_J^{(1)}\mathbf{a}&=[a(0),a(J-1),a(J-2),...,a(1)]^T \\ &=\mathbf{F}_J^{-1}\mathbf{F}_J^{\dagger}\mathbf{a} \\ &=\mathbf{F}_J^{\dagger}\mathbf{F}_J^{\dagger}\mathbf{a}, \end{aligned}$$

The former is just a time reversal of vector $\mathbf{a}$ and the latter corresponds to a single cyclic shift of time reversed vector and it also correspond to taking a J point IFFT twice of a vector $\mathbf{a}$. This case is going to be useful indeed in this paper and for practical purposes it will be just denoted as $\mathbf{P}$.

C. Cyclic Prefix Transmission

Inter symbol interference (ISI) that degrades performance of most systems usually results from long impulse response sequences often associated with frequency selective channels. In a case where one is transmitting data in blocks of symbols then the channel may affect the beginning or end of the block hence this is termed inter block interference (IBI). This problem is usually mitigated by an insertion of a guard sequence called a cyclic prefix either at the beginning or the end of each block to be transmitted. This has an effect of transforming the frequency selective channel into parallel flat fading channels.

To illustrate this effect consider a data sequence of symbols $x(n)$ that is transmitted over a frequency selective channel with impulse response $h(n)=[h(0),\ldots h(L)]^T$, where L is the channel length. The observed data sequence $y(n)$ at the receiver is given by the following convolution sum

$$y(n)=\sum_{l=0}^{L}h(l)x(n-l)+w(n), \tag{3}$$

where $w(n)$ represents noise samples which are assumed to have zero mean Gaussian with white power spectrum. From (3) it is noted that a the observed symbol at interval n is a linear combination of $L+1$ symbols and this severs as a clear demonstration of the ISI effect. Thus blocks of data are transmitted to reduces ISI however IBI still occurs since, for example, consecutive blocks with J symbols are transmitted; at the receiver the observed consecutive blocks will have corrupted symbols of at least a length of the memory of the channel i.e. $L+1$. As mentioned above this problem is resolved by inserting a CP usually by repeating the first $L+1$ entries of the block being transmitted at the end. Say now we transmit a block $\mathbf{x}$ of length J then $\mathbf{y}$ at the receiver is given by

$$\mathbf{y}=\tilde{\mathbf{H}}\mathbf{x}, \tag{4}$$

where $\tilde{\mathbf{H}}$ is a circulant with $[\tilde{\mathbf{H}}]_{p,q}=h((p-q)\bmod J)$. Circulant matrices enjoy the two following properties:

i) A circulant matrix can be diagonalized by an FFT operation.

$$\tilde{\mathbf{H}} = \mathbf{F}_J^{\dagger}\mathbf{D}(\tilde{\mathbf{h}})\mathbf{F}_J \quad \text{and} \quad \tilde{\mathbf{H}}^{\dagger} = \mathbf{F}_J^{\dagger}\mathbf{D}(\tilde{\mathbf{h}}^{*})\mathbf{F}_J \tag{5}$$

where $\mathbf{D}(\tilde{\mathbf{h}}) \equiv \mathrm{diag}(\tilde{\mathbf{h}})$ and the vector $\tilde{\mathbf{h}} \equiv [H(e^{j0}),\ldots,H(e^{j\frac{2\pi}{J}(J-1)})]^T$ has the $(p+1)$th entry being channel frequency response $H(z) \equiv \sum_{l=0}^{L} h_v(l)z^{-l}$ evaluated at the frequency $z = e^{j\frac{2\pi}{J}p}$.

ii) Pre- and Post-multiplication of a circulant matrix by **P** yields it transpose

$$\mathbf{P}\tilde{\mathbf{H}}\mathbf{P} = \tilde{\mathbf{H}}^T \quad \text{and} \quad \mathbf{P}\tilde{\mathbf{H}}^{*}\mathbf{P} = \tilde{\mathbf{H}}^{\dagger} \tag{6}$$

Now if one applies the first property to (4) then it becomes

$$\mathbf{F}_J\mathbf{y} = \mathbf{D}(\tilde{\mathbf{h}})\mathbf{F}_J\mathbf{x} \tag{7}$$

This equation shows how utilizing a CP has transformed a frequency selective channel in to a set of J parallel independent flat fading channels with fading gain of the corresponding entries of $\tilde{\mathbf{h}}$.

D. The SC-FED Transmitter

A discrete-time equivalent baseband model of a communication system with four transmit antennas and a single receive antenna is shown in Figure 1. The source feed the system with data symbols $d(n)$ which are from a set of finite alphabet $\mathcal{A}$ determined by a type of modulation scheme being used. They are passed to K×1 blocks i.e. $s(i) = [d(iK),\ldots, d(iK + K - 1)]^T$ where the serial index n is related to the block index i by $n = iK + k$, $k \in [0, K - 1]$. The QO-ST block encoder take as input four consecutive blocks $s(4i)$, $s(4i + 1)$, $s(4i + 2)$, and $s(4i + 3)$ and output the following $4J \times 4$ ST block coded matrix,

$$\begin{bmatrix} \bar{s}_1(4i) & \bar{s}_2(4i) & \bar{s}_3(4i) & \bar{s}_4(4i) \\ \bar{s}_1(4i+1) & \bar{s}_2(4i+1) & \bar{s}_3(4i+1) & \bar{s}_4(4i+1) \\ \bar{s}_1(4i+2) & \bar{s}_2(4i+2) & \bar{s}_3(4i+2) & \bar{s}_4(4i+2) \\ \bar{s}_1(4i+3) & \bar{s}_2(4i+3) & \bar{s}_3(4i+3) & \bar{s}_4(4i+3) \end{bmatrix} = \begin{bmatrix} s(4i) & s(4i+1) & s(4i+2) & s(4i+3) \\ -\mathbf{P}s^*(4i+1) & \mathbf{P}s^*(4i) & -\mathbf{P}s^*(4i+3) & \mathbf{P}s^*(4i+2) \\ s(4i) & s(4i+1) & -s(4i+2) & -s(4i+3) \\ -\mathbf{P}s^*(4i+1) & \mathbf{P}s^*(4i) & \mathbf{P}s^*(4i+3) & -\mathbf{P}s^*(4i+2) \end{bmatrix} \begin{matrix} \xrightarrow{space} \\ \downarrow time \end{matrix} \tag{8}$$

As shown in (8) the QO-ST encoder follows the following encoding rule for each of the four blocks to be transmitted.

$$\begin{aligned} \bar{s}_1(4i+1) &= -\mathbf{P}\bar{s}_2^*(4i) & \bar{s}_2(4i+1) &= \mathbf{P}\bar{s}_1^*(4i) \\ \bar{s}_1(4i+2) &= \bar{s}_1(4i) & \bar{s}_2(4i+2) &= \bar{s}_2(4i) \\ \bar{s}_1(4i+3) &= -\mathbf{P}\bar{s}_2^*(4i) & \bar{s}_2(4i+3) &= \mathbf{P}\bar{s}_1^*(4i) \\ \\ \bar{s}_3(4i+1) &= -\mathbf{P}\bar{s}_4^*(4i) & \bar{s}_4(4i+1) &= \mathbf{P}\bar{s}_3^*(4i) \\ \bar{s}_3(4i+2) &= -\bar{s}_3(4i) & \bar{s}_4(4i+2) &= -\bar{s}_4(4i) \\ \bar{s}_3(4i+3) &= \mathbf{P}\bar{s}_4^*(4i) & \bar{s}_4(4i+3) &= -\mathbf{P}\bar{s}_3^*(4i) \end{aligned} \tag{9}$$

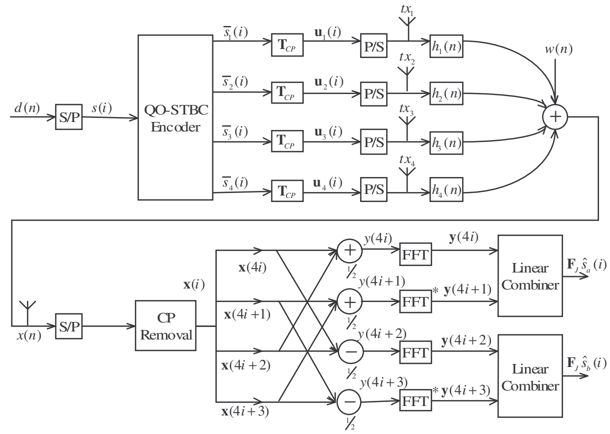

Fig. 1 SC-FED QSTBC transceiver model

As explained in Section II C to avoid IBI in the presence of a frequency-selective channel CP approach is adopted. This approach inserts a CP for each block before transmission. Mathematically at each transmit antenna $v \in [1,..,4]$ a tall P×J transmit matrix $\mathbf{T}_{CP} = [\mathbf{I}_{CP}^T, \mathbf{I}_J^T]^T$ with I_{CP} compromising the last $P - J = L$ rows of I_J is applied to $\bar{s}_v(i)$ so as to obtain P×1 blocks $\mathbf{u}_v(i) = \mathbf{T}_{CP}\bar{s}_v(i)$. When $\bar{s}_v(i)$ is multiplied by $\mathbf{T}_{CP}$ last P - J of its entries are replicated and placed on top of $\bar{s}_v(i)$

III. Receiver In An MMSE Sense

At the receiver the data symbols corresponding to the CP are discarded since they correspond to part of the block that was subjected to IBI and this results with a following observation at the receiver,

$$\mathbf{x}(i) = \sum_{v=1}^{4} \tilde{\mathbf{H}}_v \bar{s}_v(i) + \mathbf{w}(i), \tag{10}$$

where $\mathbf{w}(i)$ is an additive Gaussian noise that is assumed to be white with each entry having a variance of $\sigma_w^2 = N_0$. If now one considers four consecutive blocks (10), expanding out the sum of each of the resulting four equations and applying the encoding rule of (9) then one obtains the following interesting result;

$$\begin{aligned} \mathbf{x}(4i) = {} & \tilde{\mathbf{H}}_1\bar{s}_1(4i) + \tilde{\mathbf{H}}_2\bar{s}_2(4i) \\ & + \tilde{\mathbf{H}}_3\bar{s}_3(4i) + \tilde{\mathbf{H}}_4\bar{s}_4(4i) + \mathbf{w}(4i) \end{aligned} \tag{11}$$

$$\begin{aligned} \mathbf{x}(4i+1) = {} & -\tilde{\mathbf{H}}_1\mathbf{P}\bar{s}_2^*(4i) + \tilde{\mathbf{H}}_2\mathbf{P}\bar{s}_1^*(4i) \\ & -\tilde{\mathbf{H}}_3\mathbf{P}\bar{s}_4^*(4i) + \tilde{\mathbf{H}}_4\mathbf{P}\bar{s}_3^*(4i) + \mathbf{w}(4i+1) \end{aligned} \tag{12}$$

$$\begin{aligned} \mathbf{x}(4i+2) = {} & \tilde{\mathbf{H}}_1\bar{s}_1(4i) + \tilde{\mathbf{H}}_2\bar{s}_2(4i) \\ & -\tilde{\mathbf{H}}_3\bar{s}_3(4i) - \tilde{\mathbf{H}}_4\bar{s}_4(4i) + \mathbf{w}(4i+2) \end{aligned} \tag{13}$$

$$\begin{aligned} \mathbf{x}(4i+3) = {} & -\tilde{\mathbf{H}}_1\mathbf{P}\bar{s}_2^*(4i) + \tilde{\mathbf{H}}_2\mathbf{P}\bar{s}_1^*(4i) \\ & +\tilde{\mathbf{H}}_3\mathbf{P}\bar{s}_4^*(4i) - \tilde{\mathbf{H}}_4\mathbf{P}\bar{s}_3^*(4i) + \mathbf{w}(4i+3). \end{aligned} \tag{14}$$

As shown in Figure 1 it is now possible to decouple the above equations by performing simple linear operations; defining the result of ((11) + (13))/2 as $y(4i)$ and that of ((12) + (14))/2 as $y(4i + 1)$ then one obtains,

$$y(4i) = \tilde{\mathbf{H}}_1\bar{s}_1(4i) + \tilde{\mathbf{H}}_2\bar{s}_2(4i) + \eta(4i) \tag{15}$$

$$y(4i+1) = -\tilde{\mathbf{H}}_1\mathbf{P}\bar{s}_2^*(4i) + \tilde{\mathbf{H}}_2\mathbf{P}\bar{s}_1^*(4i) + \eta(4i+1)\,. \tag{16}$$

Similarly defining the result of ((11) - (13))/2 as $y(4i)$ and that of ((12) - (14))/2 as $y(4i + 1)$ then one obtains,

$$y(4i+2) = \tilde{\mathbf{H}}_3\bar{s}_3(4i) + \tilde{\mathbf{H}}_4\bar{s}_4(4i) + \eta(4i+2) \tag{17}$$

$$y(4i+3) = -\tilde{\mathbf{H}}_3\mathbf{P}\bar{s}_4^*(4i) + \tilde{\mathbf{H}}_4\mathbf{P}\bar{s}_3^*(4i) + \eta(4i+3)\,, \tag{18}$$

where $\eta(4i) = \frac{1}{2}(\mathbf{x}(4i) + \mathbf{x}(4i+2))$, $\eta(4i+1) = \frac{1}{2}(\mathbf{x}(4i+1) + \mathbf{x}(4i+3))$, $\eta(4i+2) = \frac{1}{2}(\mathbf{x}(4i) - \mathbf{x}(4i+2))$ $\eta(4i+3) = \frac{1}{2}(\mathbf{x}(4i+1) - \mathbf{x}(4i+3))$ the properties of this noise are the same as previously defined. It can now be noted that the system has now be decoupled in such a manor that each pair were transmitted by two antennas and are observed after four block intervals. Thus the system has a full rate which is comparable to the OSTBC case. One can now proceed are proposed in [8] for each set of equations and apply the minimum mean square-error (MMSE) estimator to obtained an estimate of the original data. One may proceed by left-multiplying (16) by **P**, conjugating and applying property *i* then one can obtain,

$$\mathbf{P}y^*(4i+1) = -\tilde{\mathbf{H}}_1^\dagger\bar{s}_2(4i) + \tilde{\mathbf{H}}_2^\dagger\mathbf{P}\bar{s}_1(4i) + \mathbf{P}\eta^*(4i+1) \tag{19}$$

The attractiveness of taking an FFT on a system with circulant matrices was alluded to in Section II C and thus established that it would be much easier to perform any further processing of the signal in frequency domain. Proceeding in this manor and duly not these definitions; $\mathbf{y}(4i) \equiv \mathbf{F}_J y(4i)$, $\mathbf{y}^*(4i+1) \equiv \mathbf{F}_J\mathbf{P}y^*(4i+1)$, $\eta(4i) \equiv \mathbf{F}_J\mu(4i)$, $\eta^*(4i+1) \equiv \mathbf{F}_J\mathbf{P}\mu^*(4i+1)$ and denote $\mathbb{D}_v \equiv \mathbf{D}(\tilde{\mathbf{h}}_v)$, $\mathbb{D}_v^* \equiv \mathbf{D}(\tilde{\mathbf{h}}_v^*)$ $v \in [1,\dots,4]$ and noting that $\mathbf{F}_J\mathbf{F}_J^\dagger = \mathbf{I}_J$ one then obtains the following,

$$\mathbf{y}(4i) = \mathbb{D}_1\mathbf{F}_J\bar{s}_1(4i) + \mathbb{D}_2\mathbf{F}_J\bar{s}_2(4i) + \eta(4i) \tag{20}$$

$$\mathbf{y}^*(4i+1) = -\mathbb{D}_1^*\mathbf{F}_J\bar{s}_2(4i) + \mathbb{D}_2^*\mathbf{F}_J\bar{s}_1(4i) + \eta^*(4i+1)\,. \tag{21}$$

One may summarise (20) and (21) as follow, and also substituting back the encoding rule of (9),

$$\breve{\mathbf{y}}_a \equiv \begin{bmatrix} \mathbf{y}(4i) \\ \mathbf{y}^*(4i+1) \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbb{D}_1 & \mathbb{D}_2 \\ \mathbb{D}_2^* & -\mathbb{D}_1^* \end{bmatrix}}_{\equiv {}^a\mathbb{D}} \begin{bmatrix} \mathbf{F}_J s(4i) \\ \mathbf{F}_J s(4i+1) \end{bmatrix} + \begin{bmatrix} \eta(4i) \\ \eta^*(4i+1) \end{bmatrix}. \tag{22}$$

Similarly if one proceeds as before then one obtains the following,

$$\breve{\mathbf{y}}_b \equiv \begin{bmatrix} \mathbf{y}(4i+2) \\ \mathbf{y}^*(4i+3) \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbb{D}_3 & \mathbb{D}_4 \\ \mathbb{D}_4^* & -\mathbb{D}_3^* \end{bmatrix}}_{\equiv {}^b\mathbb{D}} \begin{bmatrix} \mathbf{F}_J s(4i+2) \\ \mathbf{F}_J s(4i+3) \end{bmatrix} + \begin{bmatrix} \eta(4i+2) \\ \eta^*(4i+3) \end{bmatrix}. \tag{23}$$

If one is to use minimum mean square error (MMSE) equaliser then it can be easily shown that its coefficients are

$$\gamma_a = \left({}^a\mathbb{D}^\dagger\, {}^a\mathbb{D} + \frac{\sigma_w^2}{\sigma_s^2}\mathbf{I}_{2J}\right)^{-1} {}^a\mathbb{D}^\dagger \tag{24}$$

$$\gamma_b = \left({}^b\mathbb{D}^\dagger\, {}^b\mathbb{D} + \frac{\sigma_w^2}{\sigma_s^2}\mathbf{I}_{2J}\right)^{-1} {}^b\mathbb{D}^\dagger\,, \tag{25}$$

where σ_s^2 is the signal power. It follows then that the estimates for the original transmitted blocks can be written as,

$$\mathbf{F}_J\hat{s}_a(i) = \gamma_a\breve{\mathbf{y}}_a = \begin{cases} \mathbf{F}_J\hat{s}(4i) & \forall i = 4i \\ \mathbf{F}_J\hat{s}(4i+1) & \forall i = 4i+1 \end{cases} \tag{26}$$

$$\mathbf{F}_J\hat{s}_b(i) = \gamma_b\breve{\mathbf{y}}_b = \begin{cases} \mathbf{F}_J\hat{s}(4i+2) & \forall i = 4i+2 \\ \mathbf{F}_J\hat{s}(4i+3) & \forall i = 4i+3 \end{cases}. \tag{27}$$

IV. Adaptive Reciever

The above receiver requires CSI and in practical cases (as employed in this paper) a semi-blind approach is adopted where the receiver periodically receives training. Usually this period is determined by how fast the frequency-selective channel fades. If the channel is rapidly changing then retraining may need to occur more often and this decreases throughput of the system Thus the need of an adaptive receiver under such conditions becomes a necessity. The RLS algorithm provides fast tracking and, due to the special structure of STBCs, the complexity can be reduce to that of a LMS algorithm.

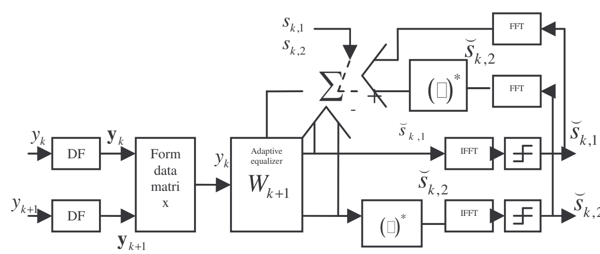

Fig. 2 Adaptive receiver for set of equations (15) and (16) [5].

Defining the linear combiner matrix of the MMSE in (26) as ${}^a\mathrm{B} \equiv \tilde{\Lambda}\Lambda^\dagger$, it can be shown that this matrix has an quadtronic structure i.e.

$${}^a\mathbf{B} = \begin{pmatrix} \mathrm{B}_1 & \mathrm{B}_2 \\ \mathrm{B}_2^* & \text{-}\mathrm{B}_1^* \end{pmatrix}$$

where

$$\mathrm{B}_1 = \mathrm{diag}\left\{\frac{1}{{}^a\mathbb{D}_0(i,i) + \sigma_w^2/\sigma_s^2}\right\}_{i=0}^{J-1} \cdot \mathbb{D}_1^*\,,$$
$$\mathrm{B}_2 = \mathrm{diag}\left\{\frac{1}{{}^a\mathbb{D}_0(i,i) + \sigma_w^2/\sigma_s^2}\right\}_{i=0}^{J-1} \cdot \mathbb{D}_2^* \tag{28}$$

and ${}^a\mathbb{D}_0 = |\mathbb{D}_1|^2 + |\mathbb{D}_2|^2$.

Thus (26) can be re-written as

$$\begin{pmatrix} \hat{s}_1^{(k)} \\ \hat{s}_2^{(k)} \end{pmatrix} = \begin{pmatrix} \mathrm{B}_1 & \mathrm{B}_2 \\ \mathrm{B}_2^* & \text{-}\mathrm{B}_1^* \end{pmatrix} \mathbf{y}_a \tag{29}$$

Rearranging (29) one obtains the form.

$$\begin{pmatrix} \hat{s}_1^{(k)} \\ \hat{s}_2^{*(k)} \end{pmatrix} = \begin{pmatrix} \mathrm{diag}(\mathbf{y}_a^{(k)}) & \mathrm{diag}(\mathbf{y}_a^{*(k+1)}) \\ \text{-}\mathrm{diag}(\mathbf{y}_a^{(k+1)}) & \mathrm{diag}(\mathbf{y}_a^{*(k)}) \end{pmatrix} \begin{pmatrix} \mathrm{E}_1 \\ \mathrm{E}_2 \end{pmatrix} = {}^a\mathbf{U}_{\mathrm{k}} E_a \quad (30)$$

The vectors E_1 and E_2 contain the diagonal entries of ${}^a\mathrm{B}_1$ and ${}^a\mathrm{B}_2$, respectively. The equalizer coefficients E_a are adaptively computed using the following recursion for every two blocks. From this point forward one can drop the *a* script that indicates that this a process for set of equations (15) and (16), this is purely for convenience.

$$E_{k+2} = E_k + Q_{k+2}\mathrm{U}_{k+2}^*\left[\mathrm{D}_{k+2} - \mathrm{U}_{k+2}E_k\right] \quad (31)$$

where

$$Q_{k+2} = \lambda^{-1}[Q_k - \lambda^{-1}Q_k\mathrm{U}_{k+2} \times\left(\mathrm{I}_{2N} + \lambda^{-1}\mathrm{U}_{k+2}Q_k\mathrm{U}_{k+2}^{\mathrm{H}}\right)^{-1}\mathrm{U}_{k+2}^{\dagger}Q_k] \quad (32)$$

The parameters of the algorithm are initialized as follows: $E_0 = 0$ and $Q_0 = \delta\mathrm{I}_{2N}$, with δ being a large number. The constant λ is called the forgetting factor and can be in the ranges between zero and unity. Using a smaller forgetting factor will result in faster tracking of the RLS algorithm. However this may result in numerical problems due the accuracy required in the algorithm. D_k is referred to as the desired response and is given by,

$$\mathrm{D}_{k+2} = \begin{cases} \begin{pmatrix} s_1^{(k+2)} \\ s_2^{*(k+2)} \end{pmatrix}, \text{for training} \\ \begin{pmatrix} \breve{s}_1^{(k+2)} \\ \breve{s}_2^{*(k+2)} \end{pmatrix}, \text{for decision-direct tracking} \end{cases} \quad (33)$$

Q_{k+2} has the following diagonal structure.

$$Q_{k+2} = \begin{pmatrix} \mathrm{Q}_{k+2} & 0 \\ 0 & \mathrm{Q}_{k+2} \end{pmatrix} \quad (34)$$

It is easy to see that the inverse term in (32) is given by a diagonal matrix

$$\left(\mathrm{I}_{2N} + \lambda^{-1}\mathrm{U}_{k+2}Q_k\mathrm{U}_{k+2}^{\dagger}\right)^{-1} = \begin{pmatrix} \Theta_{k+2} & 0 \\ 0 & \Theta_{k+2} \end{pmatrix} \quad (35)$$

The diagonal matrix Θ_{k+2} is given by

$$\Theta_{k+2} = \left(\mathrm{I}_N + \lambda^{-1}\mathrm{diag}\left(\left|\mathbf{y}_a^{(k)}\right|^2 + \left|\mathbf{y}_a^{(k+1)}\right|^2\right)\right)^{-1} \quad (36)$$

This results in Q_{k+2} having the following structure.

$$\mathrm{Q}_{k+2} = \lambda^{-1}\left(\mathrm{Q}_k - \lambda^{-1}\mathrm{Q}_k\Omega_{k+2}\mathrm{Q}_k\right) \quad (37)$$

where

$$\Omega_{k+2} = \Theta_{k+2}\mathrm{diag}\left(\left|\mathbf{y}_a^{(k)}\right|^2 + \left|\mathbf{y}_a^{(k+1)}\right|^2\right) \quad (38)$$

Thus one obtain

$$\Omega_{k+2} = \left[\mathrm{diag}\left(\left|\mathbf{y}_a^{(k)}\right|^2 + \left|\mathbf{y}_a^{(k+1)}\right|^2\right) + \lambda^{-1}\mathrm{Q}_k\right]^{-1} \quad (39)$$

Finally the RLS equalizer is then given by

$$E_{k+2} = E_k + \begin{pmatrix} \mathrm{Q}_{k+2} & 0 \\ 0 & \mathrm{Q}_{k+2} \end{pmatrix}\mathrm{U}_{k+2}^*\left[\mathrm{D}_{k+2} - \mathrm{U}_{k+2}E_k\right] \quad (40)$$

One may follow exactly the similar process for set of equations (17) and (18) to obtain the corresponding RLS adaptive receiver bearing in mind to replace the *a* scrip with *b*. Due to the diagonal structure that resulted from the special structure of the OSTBCs, the matrix inversions are in fact scalar inversions thus the receiver is not that computationally expensive. This results in an LMS complexity. The adaptive receiver for only a single set of equations is shown in Fig. 2.

V. SIMULATED RESULTS

Table 1 A typical urban (TU) channel model[5]

Delay (μsec)	0.0	0.2	0.5	1.6	2.3	5.0
Strength (dB)	-3.0	0.0	-2.0	-6.0	-8.0	-10

In the simulations the symbol rate of 271 kSymbols/s is assumed. A typical urban (TU) channel, with the power-delay profile shown in Table 1, is used along with a linearized Guassian minimum shift keying (GMSK) transmit pulse shape. The resulting channel memory is $L = 3$. All channels are assumed independent.

Defying the EGDE standard, each user has four transmit antennas with 8-PSK constellation. A Data block size of 64 symbols plus three cyclic prefix symbols is used. The algorithm is re-trained after 50 data blocks. The forgetting factor is set to 0.65.

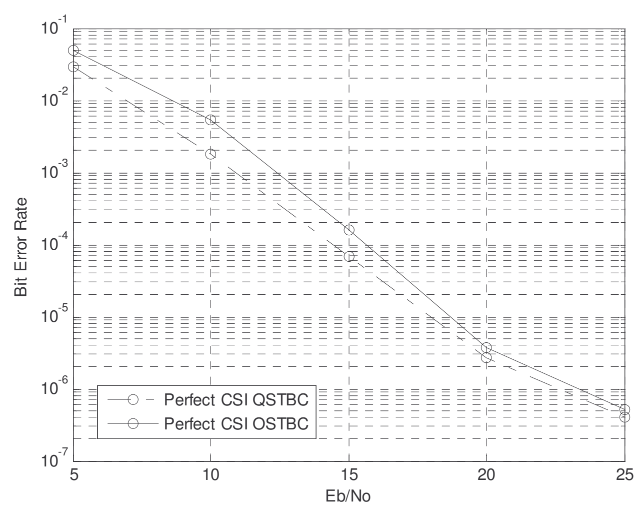

Fig.3 Comparison of QSTBC and OSTBC

Fig. 3 above compares the quasi-orthogonal STBC with four antennas to the orthogonal STBC with two antennas from [5] under perfect channel conditions. Both systems have full rate and the above result shows that a gain of at least one deci Bells is achieved by the QSTBC system at BER order of 10^{-3}.

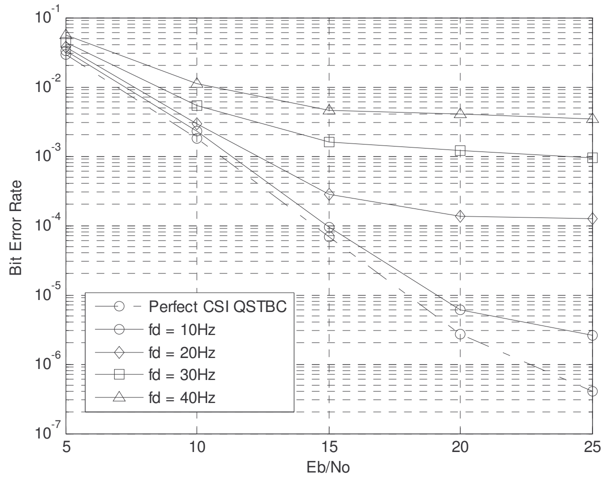

Fig.4 Effect of Doppler frequency on the performance of RLS algorithm

Fig. 3 shows the performance of the RLS algorithm at different Doppler frequencies as compared to the system with perfect CSI for the QSTBC case. From the figure it is clear that the performance of the algorithm approaches that of perfect CSI as the Doppler frequency decreases. This shows that the algorithm suffers in channels with rapid variations.

VI. Conclusion

This paper investigate the performance of an adaptive frequency-domain equalizer for quasi-orthogonal space-time block coded (QSTBC) over frequency selective fading channels. The QSTBC takes the advantage of the intrinsic orthogonal space-time block code (OSTBC) within it to reduce the complexity of the recursive least square (RLS) to that of least mean square (LMS) without compromising performance. The receiver with perfect channel state information is compare to the adaptive receiver under investigation. Also the effects of some of the parameters of the receiver are presented in the simulations. The simulation results for the OSTBC adaptive receiver are used as benchmark to compare with receiver under investigation

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