

An Optimal Pricing Model for Wireless Community Mesh Networks

Hai Ling Zhu, Wimpie Clarke, Andre Nel and M.P.F dos Santos

Abstract — The development of wireless LAN technologies offers a novel platform for internet service resale via wireless community mesh networks that provide high network coverage and lower infrastructure cost. In a wireless community mesh network, access point functions as both the Internet service provider and Internet access provider to the mesh network neighbors (end-users) since the upstream Internet service providers of the access point is not able to monitor and bill for the resold traffic within the community mesh network. In this Internet service resale business, the access provider sets their pricing policy as an Internet reseller to maximize its revenue, while the end-users who are price sensitive, respond to this pricing policy by controlling their Internet usage. Using a queuing theory model, we propose an optimal pricing model to achieve revenue maximization for a mesh network access provider. The user's sensitivity to the price is modeled in order to discover the optimal price. The effects of the price on the traffic load and the maximum number of users at the access point are explored since price is viewed as an additional strategy to encourage a better usage of the limited bandwidth resource. Monte Carlo simulation results are presented to verify the analytically optimal price based on the proposed pricing model.

Index Terms—Mesh network; Pricing; Quality of Service; Queuing model

I. INTRODUCTION

THE low-cost wireless mesh network (WMN) technology induces the expanding of wireless community mesh networks or WMNs. It is viewed as an opportunity to expand markets for telecommunication services to empower local communities and to expand economic capacity and commerce in rural areas [1].

Internet access is one of the most common applications of WMNs. In its most general form as shown in Fig. 1, a WMN interconnects stationary and / or mobile users and provides internet access as well as communications within the network. The nodes connected to the Internet are called access points (APs). In the WMN, the objectives of most end-users would be to access the Internet at a reasonable cost. In this sense, APs are Internet access providers who also resell Internet services to end-users within the WMN. The

upstream Internet service providers (ISPs) will not be able to monitor resold traffic within the WMN in order to bill the end-users. The APs set the pricing policy to generate revenue to cover their costs and maximize their profit. For the end-users as “buyers” of resold Internet services, each end-user derives some value from accessing and using the Internet, based on the APs’ pricing policy.

Each end-user’s willingness to pay for the Internet service is dependent on their perceived need for the access. Hence, it is important to analyze the end-users’ behavior when a pricing strategy is investigated to maximize revenue for an AP provider.

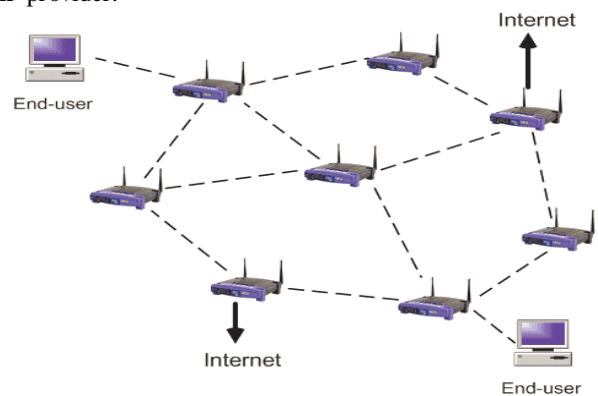


Fig.1. Mesh-based Internet Access

Quality of service (QoS) is another aspect that affects the end-users’ usage behavior. As the WMN expands, the AP whose uplink has a limited data capacity will inevitably result in the end-users having to face traffic congestion at the AP node. In general, congestion control is especially important for best-effort services, since there is no congestion avoidance mechanisms implemented in the network. Pricing is widely viewed as a mechanism to give users incentives to use the network efficiently or a means for network usage control. Using price, the network could send signals to the users, providing them with incentives that influence their usage behavior and decisions [2]. By these means, APs can provide a better and more stable service to end-users. In this paper, we will investigate how price impacts the end-user’s traffic and the system utilization at the AP node.

We model a typical mesh network in which there is a single AP providing Internet connectivity and services to the end-user nodes, as an M/M/1/S queue system [3]. The M/M/1/S is a special type of Markovian system, where customers arrive according to a Poisson process and are served by a single server with an exponential service-time distribution. The system can accommodate only S maximum customers simultaneously. In this model, as long as the end-user accepts the price charged by the AP and there is

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room in the queuing system, the end-user is served and the AP earns the revenue based on the end-user's usage.

The end-users' demand is modeled as a function of the service price. The utility function is an important concept which is widely used in literature to give a measure of the users' sensitivity to the price and their perceived QoS level.

For the general case of Internet provision, when ISPs offers a service at a particular price to the users, these users will respond to this price by changing their usage to maximize their utility. From an economic point of view, the utility function is strictly related to the user's demand curve, which is associated to the users' willingness-to-pay and their perceived QoS level.

Since it is difficult to find direct knowledge of users' utility function, we will model these dependences in our analytical pricing model instead of using a utility function to represent the end-users' demand. If the price charged by the AP is out of the range of the end-user's willingness to pay, the end-user is likely to decrease their Internet usage. It is evident that there is a trade-off between the price and the amount of end-users' Internet usage. A lower price attracts more end-users with large demands but yields less revenue per end-user, while a higher price yields more revenue per end-user but might discourage more end-users from using the service.

In this paper, an optimal pricing model to maximize the AP's revenue based on the end-users' behavior model is presented. In this pricing model, the traffic intensity, the end-users' willingness to pay and the QoS metric are taken into account to develop the optimal pricing algorithm. The proposed pricing scheme can determine an optimal price in order to maximize the revenue, while maintaining the traffic intensity and the maximum of end-users, S , at the AP to a reasonable level.

Further, we focus on usage-based pricing. Usage-based pricing is incentive compatible since it encourages customers to use network resources more efficiently [4]. The study in [5] shows that customers are willing to pay an additional per-usage charge in order to improve the network performance (QoS charge) and to avoid the performance degradation due to the network congestion.

The rest of this paper is organized as follows: In Section II, we discuss some related work about WMN pricing. In Section III, we describe a queuing system model for limited capacity systems. In Section IV, we analyze the interaction between the end-users' behavior and the price, and derive an optimization problem to determine the optimal price. In Section V, we evaluate the effects of the optimal price on the traffic load and the minimum bandwidth that the end-users can be allocated at the AP. The session level Monte-Carlo simulation results are presented to verify the analytical results. Section VI contains concluding remarks.

II. RELATED WORK

Pricing for communication networks has been researched for years, while studies in WMN pricing have only been recently performed. These studies [6,7,8] mainly focus on using pricing as an incentive to encourage participation and cooperation from self-interested nodes. In [6] the authors propose a demand and supply framework to analyze dynamic

pricing in a two-hop network with one or more service providers. Our work places the problem scenario in a community mesh network, in which cooperation between the nodes is assumed. In [7] the authors present the economic behavior of wireless nodes in wireless networks using a game theory approach. However, the authors only deal with the pricing issue under the assumption of unlimited capacity, which relies on the assumption that the wireless network channel and the AP's uplink have an unlimited capacity, or users have no minimum bandwidth requirements. We deal with a more realistic scenario in which there is a limited channel capacity at the AP and end-users are QoS sensitive. In [8] the authors analyze the pricing and purchasing strategies of the AP, relaying nodes and clients using a game theory approach and propose a pricing algorithm for the limited capacity model based on Markovian decision theory. However, our objective is to investigate the effects of the end-users' willingness-to-pay on the optimal price that maximizes the AP's revenue for a finite capacity system.

III. SYSTEM MODEL

When end-users connect to an AP to access the Internet, the AP has a total bandwidth of B (Mbits/sec) available for all the end-users. Suppose that end-users arrive at the AP according to a Poisson process distribution with an arrival rate λ (users/minute) and each end-user utilizes (transmits and receives) a certain amount of data during their connection to the Internet before disconnecting. Literature on data analysis of internet traffic describes the sizes of the files transmitted over the Internet as being heavy tail distributed. Furthermore, in [9] the authors show that the transmission durations also follows a heavy-tailed distribution due to the heavy-tailed distributed file sizes. It is generally assumed that the service time is strongly correlated to the file size. In this context, the service time is taken to be equal to the transmission time of a file, which is proportional to the size of the file [10]. For simplification, we suppose that file sizes are exponentially distributed with mean F (Mbits). When there are $N \geq 1$ users in the system at the same time, each user is allocated an instantaneous bandwidth of B/N . This system as shown in Fig.2 is equivalent to an M/M/1/S PS (Processor Sharing) queue with arrival rate λ and exponentially distributed service rate μ (minute). Then the traffic intensity is denoted as $\rho = \lambda / \mu$. In practice, it is suggested that a shared resource should be designed in such a way that its utilization is less than two-third of its full capacity ($\rho=1$) [3].

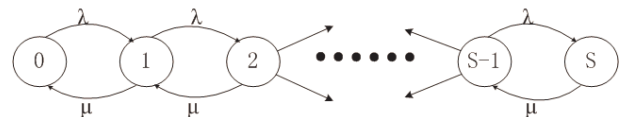


Fig.2. M/M/1/S model

IV. PRICING MODEL

From [3], in an M/M/1/S queue system, when there are S customers in the system or the system is saturated, the blocking probability P_s is:

$$P_s = \frac{(1-\rho)\rho^S}{1-\rho^{S+1}}. \quad (1)$$

Since the maximum value of end-users that can be accommodated (S) is determined by the blocking probability given a certain traffic intensity ρ , the minimum bandwidth allocation for a certain P_s is B/S given a certain traffic intensity ρ . Therefore, we can also say a minimum bandwidth allocated to the end-users can be determined by a given P_s . From the viewpoint of the AP providers, the lower blocking probability (P_s) implies that more end-users can obtain services and more revenue can be generated. However, according to Eq.1 the lower blocking probability also means that the maximum of end-users (S) in the system is higher. Hence, from the end-user point of view, even while their requests can be accommodated with higher probability due to the lower blocking probability, the minimum bandwidth that can be allocated to them might be lower. In this pricing model, we use the blocking probability as an indicator of the minimum bandwidth to determine the limitation for the value of P_s . In Section V, we will use the blocking probability to model the end-users' reaction to the minimum bandwidth.

According to queue theory, the average number of customers at the services facility N_s is:

$$N_s = \rho(1-P_s) \quad (2)$$

However, the arriving end-users are price sensitive. Their responses to the price charged by the AP depend on a number of factors. In [11] the authors suggest that a probabilistic model for end-user's willingness-to-pay the quoted price using a Pareto distribution of customer capacity to pay. Every end-user has the capacity to pay based on a Pareto distribution with scale b and shape α , where all customers have capacities at least as large as b and α determines how the capacities are distributed. Thus α and b are the Pareto distribution parameters for the end-user capacity to pay function. It is reasonable to assume that end-users' willingness-to-pay is associated with their capacities to pay. Therefore, the expectation of acceptance given price p is:

$$E = \begin{cases} 1 - \frac{\alpha}{\alpha+\delta} \left(\frac{p}{b}\right)^\delta, & 0 \leq p \leq b \\ \frac{\alpha}{\alpha+\delta} \left(\frac{b}{p}\right)^\delta, & p > b \end{cases} \quad (3)$$

Where δ is the equivalent to the economic elasticity of demand of the end-users. The higher the value of δ , the more willing the end-users is to pay.

Thus, the "arrival rate" of our model is different from that of the conventional M/M/1/S queuing system as mentioned above by a factor of E : λE . In other words, the arrival rate is denoted as a function of the price.

Correspondingly, the traffic intensity of the system model that we mentioned above is denoted as $\rho(p) = \lambda E(p) / \mu$. Fig.3 shows that the traffic intensity decreases when the quoted price increases, since a higher price leads to a smaller end-user arrival rate. Substituting $\rho(p)$ for ρ in (1) and (2), we get the following:

$$P_s = \frac{(1-\rho(p))\rho(p)^S}{1-\rho(p)^{S+1}} \quad (4)$$

$$N_s = \rho(p)(1-P_s) \quad (5)$$

Therefore, the long-term average revenue per unit time is expressed as:

$$\begin{aligned} AvR(p) &= N_s * F * p \\ &= \rho(p) * (1-P_s) * F * p \end{aligned} \quad (6)$$

where p is the price per Mbit of data transferred.

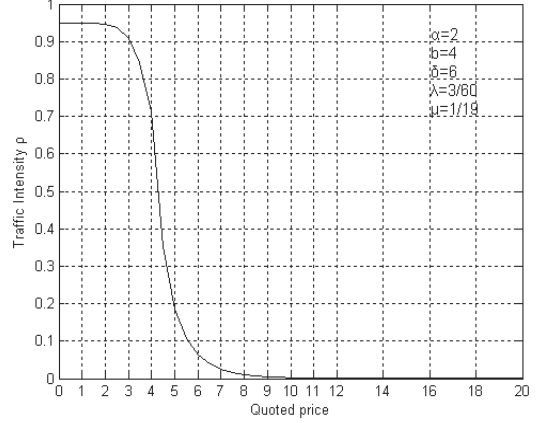


Fig.3. Traffic intensity versus quoted price

Now we can formulate the optimization problem to determine the optimal price:

$$\begin{aligned} &\text{maximize } AvR(p) = \rho(p) * (1-P_s) * F * p \\ &\text{subject to } P_s \leq \varepsilon, S \geq 0 \end{aligned} \quad (7)$$

where ε is a constraint of blocking probability P_s . The solution of this optimization problem is characterized by

$$\frac{\partial AvR(p)}{\partial p} = \rho(p) + p \frac{\partial \rho(p)}{\partial p} = 0. \quad (8)$$

Therefore, the long-term average revenue per unit time is

maximized when p_{opt} is equal to $\left(\frac{\alpha+\delta}{(\delta+1)\alpha}\right)^{\frac{1}{\delta}} b$.

V. SIMULATION

In this section, we show some simulation results to evaluate and validate the analytical model discussed in the previous section. Taking into account the usage of pricing as a traffic control mechanism, we choose a value close to one as initial traffic intensity ρ to investigate the traffic load at the AP. As the load approaches the full capacity of the system ($\rho=1$), the number of customers grows rapidly without bound and the system becomes unstable [3].

The objective function in Eq.7 is shown numerically in Fig.4 and Fig.5 with $\alpha=2$, $b=4$, $\delta=6$, $\varepsilon=0.01$. Fig.5 and Fig.6 characterizes the relationships between the revenue, the price and the end-users' response to the price by adjusting the amount of usage of the Internet service. As the price increases, the number of the end-users decreases while the revenue per end-user increases. The total revenue is maximized when the price reaches the optimal price p_{opt} .

It is shown in Fig.4 and Fig.5 that as the quoted price reaches the optimal price p_{opt} , the maximum number of customers S reduces from 34 to 15, which indicates that the minimum bandwidth allocation for the end-users increases, and the traffic intensity dropped from 0.95 to 0.84 which is

approximately in accordance with the “two-third” rule as mentioned in Section III.

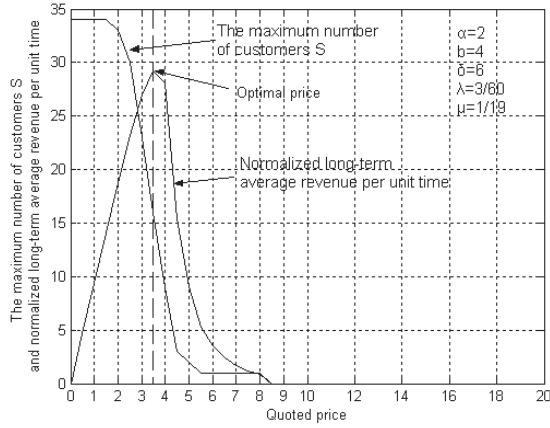


Fig. 4. Normalized long-term average revenue per unit time and the maximum number of customer S versus quoted price.

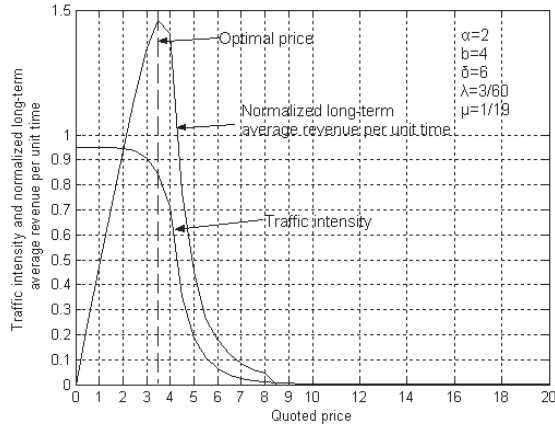


Fig. 5. Long-term average revenue per unit time and traffic intensity versus quoted price.

Fig.6-Fig.8 shows how the parameters of the function of willingness-to-pay affect the value of the optimal price. It is shown that the parameters α and δ have little impact on the value of the optimal price while parameter b plays an important role in determining the optimal price.

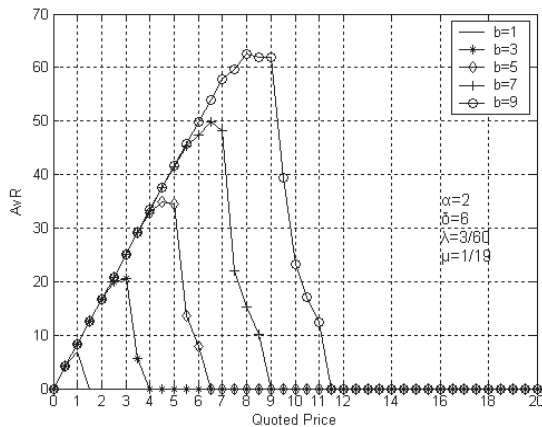


Fig. 6. Long-term average revenue per unit time versus quoted price for different b .

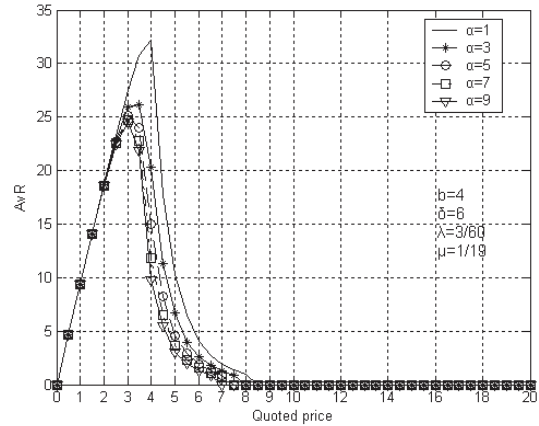


Fig.7. Long-term average revenue per unit time versus quoted price for different α .

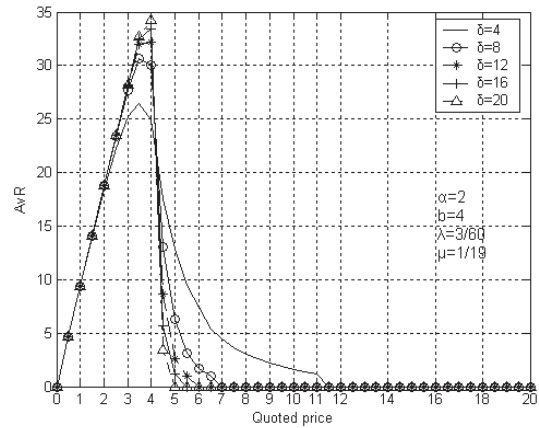


Fig.8. Long-term average revenue per unit time versus quoted price for different δ .

The AP provider has no knowledge about the parameters α , b and δ of the expectation of acceptance E in Eq.3. Moreover it is impossible for the AP providers to find related information by means of observations or interviews, because the typical end-user does not have a satisfactory benchmark with which to compare prices for what is to him/her an abstract quantity (i.e. 1Mbit data). However, the AP provider can observe the end-users’ acceptance to the quoted price online. In [11] the authors suggest an adaptive algorithm to learn these parameters from the observed acceptance rate that end-users accept a given price. Since the optimal price is not very sensitive to α and δ , one only needs to adjust the estimated value b based on the real-time user reaction. This will make the implementation of this optimal pricing scheme easier. In fact, the process of learning these parameters is a dynamic process with the aim to make the price closer to the end-users’ real perception about the QoS, since end-users’ evaluation about the Internet service can fluctuate due to various internal and external factors. The objective of dynamically learning these parameters is to capture the time-varying feature of customer behavior.

We further assume that the end-users are sensitive to the changes of the minimum bandwidth provided. According to Eq. 1, S is reduced as blocking probability decreases given a certain traffic intensity ρ . Obviously, it is reasonable to assume that a higher minimum bandwidth encourages users

to spend more time on the Internet. For simplification, we will use the blocking probability P_s as an indicator of the minimum bandwidth as we mentioned in Section IV. Here we use the following function to model the mean service time μ 's dependence on the blocking probability P_s :

$$\mu(P_s) = \mu e^{-P_s \cdot \beta} \quad (8)$$

where β is a factor related to the end-users' behavior. The higher the value of β is, the less sensitive the user is to the minimum bandwidth. Therefore, the traffic intensity can be written as $\rho(P_s) = \lambda E / \mu(P_s)$. Fig.9 illustrates that the traffic intensity ρ increases with the increasing blocking probability P_s . This is because the mean service time μ decreases. Note the fact that the traffic intensity is greater than one indicates that customers arrive faster than they are served and the system becomes unstable. Therefore, the value of P_s must be smaller than 0.067 as shown in Fig.10.

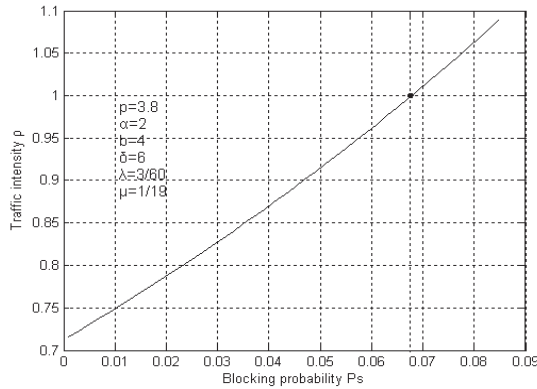


Fig.9. Traffic intensity versus blocking probability.

Using the M/M/1/S PS queue system model described in Section III, a session level Monte-Carlo simulation is performed to validate the analytical pricing model. In the Monte-Carlo simulation, the optimal price can be found by an exhaustive search. In Fig.10, the aggregated revenue of the Monte-Carlo simulation and the analytic long-term average revenue per unit time are plotted over the quoted price to compare the optimal prices for the Monte-Carlo simulation and the analytical model. It is shown that the optimal price for the Monte-Carlo simulation, which is equal to 4, is close to the theoretical optimal price p_{opt} (3.644).

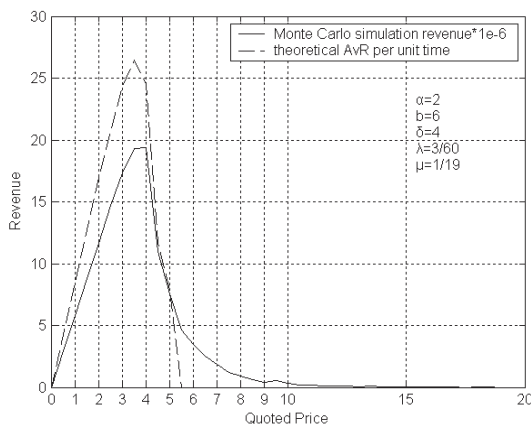


Fig.10. Simulation- versus theoretical revenue.

In Figs.11-13, all curves comparing the Monte Carlo simulation results to the analytic results are given in terms of the different parameters of end-user's willingness-to-pay. We observe that only the parameter b plays a determining role in the optimal price as mentioned above.

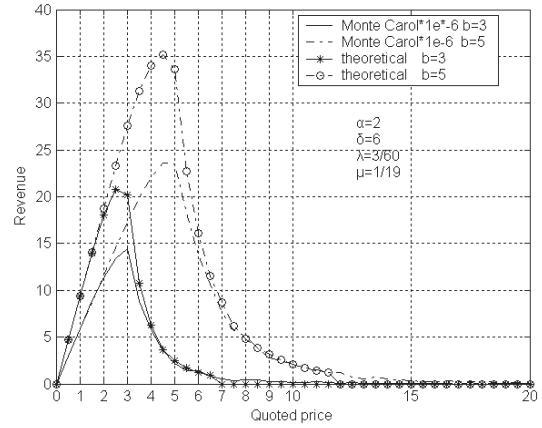


Fig.11. Simulation- versus theoretical: revenue per unit time versus quoted price for different b .

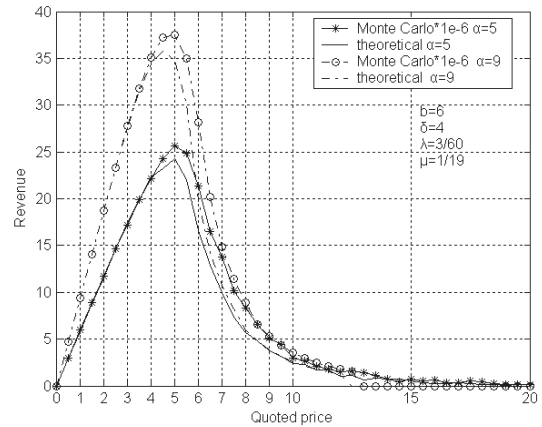


Fig.12. Simulation- versus theoretical: revenue per unit time versus quoted price for different α .

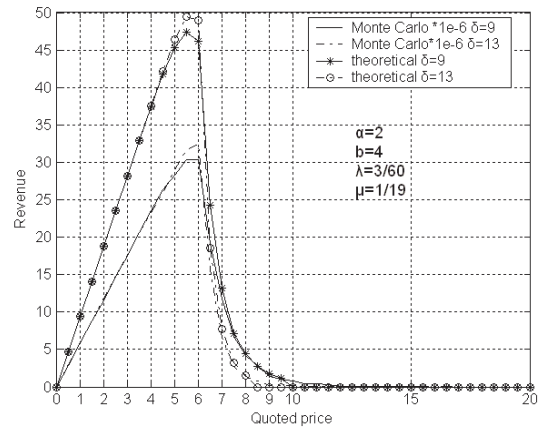


Fig.13. Simulation- versus theoretical: revenue per unit time versus quoted price for different δ .

VI. CONCLUSION AND FUTURE WORK

In this paper, an optimal pricing model for Internet service resale via WMN was presented. We model end-users' traffic as a function of the service price. Based on a queuing system model, an optimal price algorithm is developed to maximize the AP providers' revenue. The performance of the proposed pricing scheme is investigated and the analytically optimal price is compared to the results of the session level Monte-Carlo simulations. It was shown that the proposed optimal pricing scheme can provide maximized revenue to the AP provider, a better quality of service in terms of the minimum bandwidth allocation to the end-users and an efficient control of the traffic load at the AP node to avoid congestion.

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