A Channel and Delay Estimation Algorithm for Asynchronous Cooperative Diversity with Pilot Symbol Design

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Abstract—This paper serves as a simple extension to an already proposed scheme for channel and delay estimation for an asynchronous cooperative diversity network in a Rayleigh Block-Flat-Fading channel. Pilot symbol design is also discussed for M-PSK modulating schemes. It is also shown that the Cramer-Rao lower bound for channel and delay estimation is achieved for different delay values.

Index Terms—Asynchronous Cooperative Diversity, Cyclic Prefix, Distributed Space-Time Coding, Pilot Symbol Design.

I. INTRODUCTION

IT is a well known fact that multiple antennas at the receiver and transmitter in Multiple-Input-Multiple-Output (MIMO) systems provide spatial diversity which tremendously helps to combat the negative effects of fading in wireless communication channels. But it is also a well known fact that there is an increasing demand for wireless devices to be smaller and have lower power consumptions, making it impractical to have multiple antennas on a mobile device, thus the birth of Cooperative Communication. In [1] it is shown that even with a noisy interuser channel, cooperative communication still leads to improved performance and a generally more robust system. But with the benefits of Cooperative Diversity come the disadvantage of asynchronisity, and the challenge therein lies to estimate the relative delay between the transmitters in the system. In [2] this problem was addressed using modified space-time block codes, but at the expense of implementation practicality. In [3] however, a simple channel and delay estimation algorithm is proposed that achieves the Cramer-Rao bound (CRB). The motivation behind this paper is to extend on that scheme, providing a simple extension to the algorithm to allow for negative delays i.e. the data from transmitter one arriving at the receiver *after* the data from transmitter two. In [3] only positive delays are considered. There is also a section discussing the design of pilot symbols that should be used.

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This paper is arranged as follows: In section II the system model is given, in section III the channel and delay estimation algorithm is explained, in section IV the detection scheme for a single receiver is shown, in section V the extensions required for negative delays are highlighted, in section VI the design of pilot symbols is discussed and section VII is for the simulation results.

Lower case boldface letters indicate vectors while upper case boldface letters indicate matrices. tr(**A**) is the trace of matrix **A**, $(\ )^*$, $(\ )^H$, $(\ )^\#$ and $|\ |$ are the conjugate, the hermitian, the pseudo-inverse and absolute value operators respectively. E[] is the expectation operator, $\mathbf{I}_N$ and $\mathbf{O}_N$ are N x N identity and zero matrices respectively and $\mathbf{o}_N$ is an N x 1 vector of zeros . $\mathbf{A}_{N\times M}$ is a matrix with N rows and M columns. $\mathbf{a}(n)$ is the n^{th} block of symbols and $\mathbf{a}(n, k)$ is the k^{th} element in $\mathbf{a}(n)$.

II. SYSTEM MODEL

A channel and delay estimation algorithm for an asynchronous cooperative diversity system using a linear block precoding with a training sequence as cyclic prefix is considered. The system model is a macrocell, specifically two base stations transmitting the same data to one mobile node. Each base station and mobile node has only one transmit or receive antenna. This virtual MIMO link allows for the exploitation of cooperative diversity. The symbols to be transmitted are replicated in *space* and *time* according to Alamouti's scheme [4], allowing the mobile node to combine and decode the two received signals using a simple linear technique to reap the benefits of diversity.

We start off with the block of symbols that is to be transmitted being parsed into two sub-blocks of N symbols each, $\mathbf{d}(n)$ and $\mathbf{d}(n + 1)$. Training symbols $\mathbf{d}_1$ and $\mathbf{d}_2$ each of length L are then added at the end of these sub-blocks to form two $(N + L)$ x 1 vectors. To insert a cyclic prefix of training symbols between any two successive blocks we pre-multiply these vectors with a precoding matrix $\mathbf{F}_p$ which results in two $(N + 2L)$ x 1 vectors $\mathbf{s}(n)$ and $\mathbf{s}(n + 1)$ which are represented in Fig. 1.

Fig. 1. A Frame description

The frames are then transmitted according to the block form of Alamouti's scheme given by

$$\mathbf{s}_1[n] = \begin{bmatrix} \mathbf{s}(n) \\ -(\mathbf{Ts}(n+1))^* \end{bmatrix} \quad (1)$$

$$\mathbf{s}_2[n] = \begin{bmatrix} \mathbf{s}(n+1) \\ (\mathbf{Ts}(n))^* \end{bmatrix} \quad (2)$$

and also shown in Table 1.

TABLE 1
TRANSMISSION SCHEME

	n^{th} *block symbols*	$(n+1)^{th}$ *block symbols*
$\mathbf{s_1}[n]$	$\mathbf{s}(n)$	$-(\mathbf{Ts}(n+1))^*$
$\mathbf{s_2}[n]$	$\mathbf{s}(n+1)$	$(\mathbf{Ts}(n))^*$

The precoding matrix $\mathbf{F}_p = \begin{bmatrix} \mathbf{O}_{L\times N} & \mathbf{I}_L \\ \mathbf{I}_{N+L} & \end{bmatrix}$

and the time-reversal matrix $\mathbf{T}(k, N+2L+1-k) = 1$, k = 1,….,($N$ + 2L) are represented in Fig. 2.

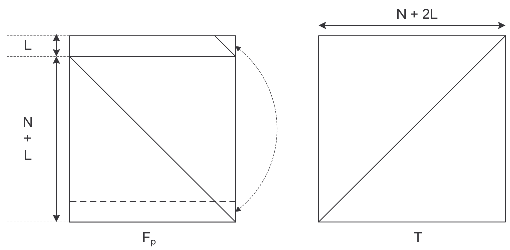

Fig. 2. Precoding matrices

If we define τ_1 and τ_2 as the arrival time of the first and second signals respectively, we can then have the received signal given by

$$\mathbf{r} = \mathbf{A}(\tau)\mathbf{Xh} + \mathbf{b} \quad (3)$$

where

$$\mathbf{X} = \begin{bmatrix} \mathbf{s}_1[n-1] & \mathbf{o}_{2N+4L} \\ \mathbf{s}_1[n] & \mathbf{o}_{2N+4L} \\ \mathbf{o}_{2N+4L} & \mathbf{s}_2[n-1] \\ \mathbf{o}_{2N+4L} & \mathbf{s}_2[n] \end{bmatrix} \quad (4)$$

$$\mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \quad (5)$$

$$\mathbf{A}(\tau) = \begin{bmatrix} \mathbf{O}_{2N+4L} & \mathbf{I}_{2N+4L} & \mathbf{\Gamma} & \mathbf{\Psi} \end{bmatrix} \quad \text{for } \tau_1 \le \tau_2 \quad (6)$$

or

$$\mathbf{A}(\tau) = \begin{bmatrix} \mathbf{\Gamma} & \mathbf{\Psi} & \mathbf{O}_{2N+4L} & \mathbf{I}_{2N+4L} \end{bmatrix} \quad \text{for } \tau_1 > \tau_2 \quad (7)$$

In the equation above h_i, i =1,2 are the complex scalar channel parameters with zero mean and a variance of 1, where the channel is assumed to be Rayleigh block-flat-fading and constant over all four blocks shown in Table 1. **b** is the zero mean complex additive white Gaussian noise vector with each entry having a variance of N_0. **X** is the data matrix obtained by stacking two consecutive frames from each transmitter. **Γ** and **Ψ** account for the asynchronism between the two signals and are given by

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{O}_{|L_\tau|\times(2N+4L-|L_\tau|)} & \mathbf{I}_{|L_\tau|} \\ \mathbf{O}_{(2N+4L-|L_\tau|)\times(2N+4L-|L_\tau|)} & \mathbf{O}_{(2N+4L-|L_\tau|)\times(|L_\tau|)} \end{bmatrix} \quad (8)$$

$$\mathbf{\Psi} = \begin{bmatrix} \mathbf{O}_{|L_\tau|\times(2N+4L-|L_\tau|)} & \mathbf{O}_{|L_\tau|} \\ \mathbf{I}_{2N+4L-|L_\tau|} & \mathbf{O}_{(2N+4L-|L_\tau|)\times(|L_\tau|)} \end{bmatrix} \quad (9)$$

where $L_\tau = \tau_2 - \tau_1$ is the relative delay between the two signals and is bounded by L. It is worthy to note that for the synchronous case, $\mathbf{\Gamma} = \mathbf{0}_{2N+4L}$ and $\mathbf{\Psi} = \mathbf{I}_{2N+4L}$.

III. CHANNEL AND DELAY ESTIMATION ALGORITHM

A maximum likelihood (ML) method for channel and delay estimation is used. We denote

$$\mathbf{ts}_1 = \mathbf{T}_s\mathbf{d}_1 \quad (10)$$

$$\mathbf{ts}_2 = \mathbf{T}_s\mathbf{d}_2 \quad (11)$$

where $\mathbf{T}_s$ is a square time reversal matrix of size L.

We then define $\mathbf{S}(\tau) = [\mathbf{ss}_1\ \mathbf{ss}_2]$ where τ is an estimate of L_τ.

If $\tau \ge 0$

$$\mathbf{ss}_1 = \begin{bmatrix} \mathbf{d}_1(\tau+1:L) \\ -(\mathbf{ts}_2)^* \end{bmatrix} \quad (12)$$

$$\mathbf{ss}_2 = \begin{bmatrix} \mathbf{d}_2 \\ (\mathbf{ts}_1(1:L-\tau))^* \end{bmatrix} \quad (13)$$

else for $\tau < 0$

$$\mathbf{ss}_1 = \begin{bmatrix} \mathbf{d}_1 \\ -(\mathbf{ts}_2(1:L-|\tau|))^* \end{bmatrix} \quad (14)$$

$$\mathbf{ss}_2 = \begin{bmatrix} \mathbf{d}_2(|\tau|+1:L) \\ (\mathbf{ts}_1)^* \end{bmatrix} \quad (15)$$

.

For this deterministic model, we denote $\mathbf{z}(\tau) = \mathbf{r}[n, N + L + |\tau| + 1: N + 3L]$. For a clearer understanding, $\mathbf{S}(\tau)$ is shown for the case $\tau_1 \le \tau_2$ in Fig. 3.

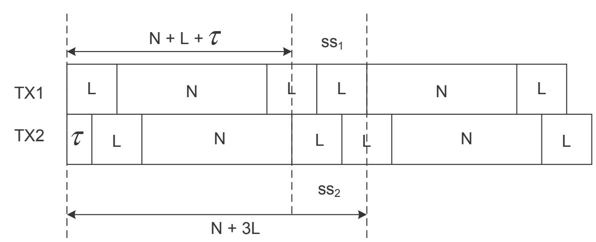

Fig. 3. Description of $\mathbf{S}(\tau)$

It follows that $\mathbf{z}(\tau) = \mathbf{S}(\tau)\mathbf{h}$, therefore

$$\tilde{\mathbf{h}}(\tau) = (\mathbf{S}(\tau))^{\#}\mathbf{z}(\tau) \quad (16)$$

where $\tilde{\mathbf{h}}(\tau)$ is an estimate of **h** based on a given value of τ.

The maximum likelihood estimator is given by

$$\hat{\tau} = \arg\min_{|\tau|<L} \left\| \mathbf{z} - \mathbf{S}(\tau)\tilde{\mathbf{h}}(\tau) \right\|^2 \quad (17)$$

and

$$\hat{\mathbf{h}} = (\mathbf{S}(\hat{\tau}))^{\#}\mathbf{z}(\hat{\tau}) \quad (18)$$

where $\hat{\mathbf{h}}$ is the estimate of **h,** based on the estimate $\hat{\tau}$.

The Mean Square Error (MSE) of the channel estimation is defined by

$$MSE(h) = \mathrm{E}\left[\left\| \mathbf{h} - \hat{\mathbf{h}} \right\|^2 \right] \quad (19)$$

The Cramer-Rao bound (CRB) for this model as derived in

[3] is given by

$$CRB = tr\left(N_0 \left[(\mathbf{S}(L_\tau))^H \mathbf{S}(L_\tau) \right]^{-1} \right) \quad (20)$$

IV. Detection Scheme

We define $\mathbf{r}_a$, $\mathbf{r}_b$ and $\mathbf{y}$ as

$$\mathbf{r}_a = \mathbf{r}\left[n, L+1 : N+L+\left|\hat{\tau}\right| \right] \quad (21)$$

$$\mathbf{r}_b = \mathbf{r}\left[n, N+3L+1 : 2N+3L+\left|\hat{\tau}\right| \right] \quad (22)$$

$$\mathbf{y} = \left[\mathbf{r}_a{}^T \quad \mathbf{r}_b{}^T \right] \quad (23)$$

The following two signals are sent to a standard SISO equalizer:

if $\hat{\tau} \geq 0$

$$\tilde{\mathbf{d}}(n,k) = \hat{h}_1^*(n)\mathbf{y}(n,k) + \hat{h}_2(n)\mathbf{y}^*(n,l+1-k) \quad (24)$$

$$\tilde{\mathbf{d}}(n+1,k) = \hat{h}_2^*(n)\mathbf{y}(n,\hat{\tau}+k) - \hat{h}_1(n)\mathbf{y}^*(n,l-\hat{\tau}+1-k) \quad (25)$$

else

$$\tilde{\mathbf{d}}(n,k) = \hat{h}_1^*(n)\mathbf{y}(n,\left|\hat{\tau}\right|+k) + \hat{h}_2(n)\mathbf{y}^*(n,l+\left|\hat{\tau}\right|+1-k) \quad (26)$$

$$\tilde{\mathbf{d}}(n+1,k) = \hat{h}_2^*(n)\mathbf{y}(n,k) - \hat{h}_1(n)\mathbf{y}^*(n,l+1-k) \quad (27)$$

where l is the length of $\mathbf{y}$.

The above equations essentially implement the inverse of Alamouti's scheme.

V. Extension for Negative Delays

As mentioned earlier, the original scheme was derived under the assumption that transmitter one's data arrives before transmitter two's data, i.e. $\tau \geq 0$. The objective of this section is to relax that assumption and to explain the simple extension on the original scheme to accommodate for the scenario when $\tau < 0$.

By definition, $\mathbf{S}(\tau)$ is the section of the signal where the pilot symbols overlap (see section VI). For the case when $\tau < 0$, (12) and (13) do not describe $\mathbf{S}(\tau)$ correctly and hence will produce erroneous channel and delay estimates. Also, (24) and (25) will not send the correct section of the received signal to the linear equalizer. The extension that is needed is that (12), (13), (24) and (25) will have to be replaced by (14), (15), (26) and (27) respectively when $\tau < 0$. Also, the maximum likelihood receiver in (17) will now have to search from -(L-1) to (L-1).

VI. Pilot Symbol Design

Much has been mentioned in the past about the *placement* of pilot symbols in a frame, and it was determined that placing a cluster of pilot symbols at the edges of a frame yielded the lowest CRB [5]. But not much has been said about what actual symbols should be used for the pilot symbols to obtain a competent system. We will attempt to present the reader with some guidelines for pilot symbol selection, but first we will explain how the pilot symbols are used in this particular system.

From section III it should be noted that $\mathbf{S}(\tau)$ is the section of the signal that contains *only* pilot symbols. Fig. 4 shows how $\mathbf{S}(\tau)$ varies as τ varies.

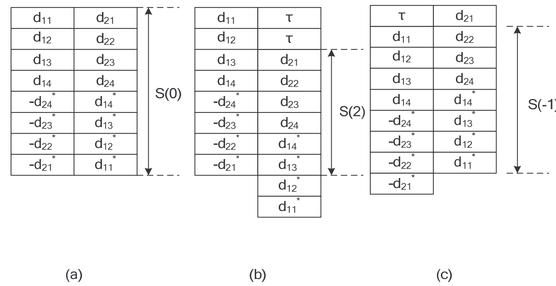

Fig. 4. Examples of $\mathbf{S}(\tau)$ with L = 4 and (a) τ = 0, (b) τ = 2 and (c) τ = -1.

As can be seen from Fig. 4, the pilot symbols from TX1 and TX2 merely slide over each other as τ varies, and $\mathbf{S}(\tau)$ is the section of the signal where the pilot symbols overlap.

The first criterion for pilot symbol design is that of orthogonality [5]-[7], i.e. the training sequence from transmitter one must be orthogonal to the training sequence from transmitter two. The reason being that it minimizes the CRB with respect to channel estimation and negates inter-symbol interference. With regard to delay estimation, it is desired that the out-of-phase auto-correlation terms of each training sequence be as small as possible [7]. As to be expected, for superior delay estimation we would desire the highest correlation to occur when the training sequence is in-phase with itself.

VII. Simulation Results

The simulator used was MATLAB version 7. Each frame (as described by Fig. 1) contains 144 symbols of which 112 are for data. Therefore the cyclic prefix contains (2 × 16) pilot symbols. 4-PSK modulation was used, and the channel and noise parameters used were as described from (3). Each combination of L_τ and signal to noise ratio (SNR) was simulated for 5000 frames. Fig. 6 shows the MSE of the channel estimation for different values of L_τ. As can be seen, the MSE of the channel estimation for each L_τ overlaps with its corresponding CRB. Since the CRB is a fundamental lower bound on the performance of any unbiased estimator, the estimation technique being used is optimal with respect to delay estimation. Also, it can be seen that the estimation performance decreases for larger delays, which displays $\mathbf{S}(L_\tau)$'s effect on (20).

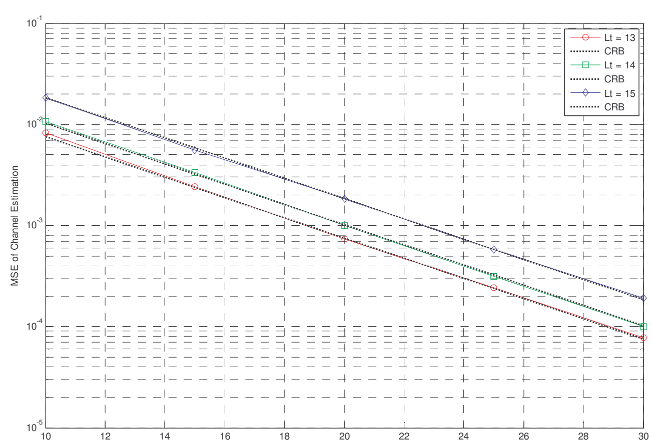

Fig. 6. Performance of the channel estimation for different values of L_τ

VIII. Conclusion

In this paper we reviewed and extended a simple channel and delay estimation algorithm and showed how the pilot symbols were used for this estimation. We also discussed what simple steps should be taken when choosing appropriate pilot symbols. This work is also important as it serves as a stepping stone to future intended work to be undertaken by the author on channel and delay estimation for an asynchronous cooperative diversity network in a frequency-selective fading channel while also implementing distributed single-carrier space-time block coding. Basically the author will attempt to take the scheme presented in this paper which is derived for frequency-flat fading channels and extend it to accommodate for frequency-selective channels. Although this presents itself as a far more complex estimation problem, as more channel coefficients need to be estimated, the author will attempt to negate the effects of the increase in overall estimation error by reaping the diversity benefits of this frequency-selectivity.

References

[1] A. Sendonaris, E. Erkip, B. Aazhang, "User Cooperation Diversity - Part I: System Description," *IEEE Trans. on Communications*, vol. 51, no. 11, pp. 1927 – 1938, Nov 2003.

[2] P. Elia, S. Kittipiyakul, T. Javidi, "Cooperative diversity schemes for asynchronous wireless networks," *Wireless Personal Communications*, vol. 43, issue 1, pp. 3-12, Oct 2007.

[3] K. Tourki, L. Deneire, "Channel and Delay Estimation Algorithm for Asynchronous Cooperative Diversity," *Wireless Personal Communications*, vol. 37, issue 3-4, pp. 361-369, May 2006.

[4] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 16, pp. 1451-1458, Oct 1998.

[5] M. Dong, L. Tong, "Optimal Design and Placement of Pilot Symbols for Channel Estimation" *IEEE Trans. on Signal Processing,* vol. 50, issue 12, pp. 3055-3069, Dec 2002.

[6] X. Ma, L. Yang, G.B. Giannakis, "Optimal training for MIMO frequency-selective fading channels" *IEEE Trans. on Wireless Communications,* vol. 4, issue 2, pp. 453 - 466, Mar 2005.

[7] Y.C. Wu, S.C. Chan, "Symbol-Timing Estimation in Space-Time Coding Systems Based on Orthogonal Training Sequences" *IEEE Trans. on Wireless Communications,* vol. 4, issue 2, pp. 603 - 613, Mar 2005.

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