Carrier Frequency Offset Estimation in WCDMA Systems Using a Modified FFT-Based Algorithm

Seare H. Rezenom and Anthony D. Broadhurst, *Member, IEEE*

Abstract**-- Wideband Code Division Multiple Access (WCDMA) is one of the wireless access technologies that delivers high data rates for the third generation communication systems. It uses different scrambling codes to differentiate among base stations. The receiving terminal needs to decode the correct identity of the scrambling code. The Fast Fourier Transform (FFT) technique is used to estimate the carrier frequency offset in a WCDMA system. This paper discusses the FFT based estimation technique and proposes a new method that employs overlapping partition sequences of the FFT window. The proposed method is compared to the conventional non-overlapped algorithm.**

I. INTRODUCTION

Wideband Code Division Multiple Access (WCDMA) is one of the radio access technologies to deliver high data rate communications systems [1]. In such systems, different scrambling codes are used to differentiate between base stations. Each base station is identified by a unique scrambling code sequence. Gold codes are normally used for this purpose [1] due to their good auto and cross-correlation properties. The process whereby a mobile station searches, synchronises and locks to a nearby base station is called cell search [2].

The process of cell search is affected by a number of factors. One of these is the presence of the frequency offset of the local oscillator at the receiver caused by the voltage controlled oscillator at the receiver oscillating at a different frequency to that of the transmitter [3]. This difference is seen as a frequency offset at the receiver and is a major cause of mobile communications failure [3]. The process of searching for a particular cell-specific scrambling code is done with algorithms that are robust to the presence of frequency offset [2][4]. These algorithms are generally shown to be reliable to detect and synchronise to a base station with frequency offsets of up to 20 kHz.

Most commercial crystal oscillators have a frequency accuracy of 3 – 13 ppm [2]. In the operating frequency range of WCDMA systems (2 GHz), this translates to a frequency offset range of 6 – 26 kHz. A mobile receiver then has to face this frequency offset before the data decoding process can begin. The Third Generation Partnership Project (3GPP) standard allows for a maximum frequency offset of 200 Hz at the receiver for reliable data recovery [1]. This constraint highlights the need to develop efficient algorithms to estimate and reduce the offset to within this specification.

Frequency offset estimation in WCDMA systems can be broadly classified to fall into three categories; FFT-based algorithms, phase increment-based algorithms and autocorrelation-based algorithms. FFT-based algorithms are discussed in [2][5]. They use the FFT of the de-spread signal to estimate the frequency offset. Phase increment based algorithms exploit the phase difference between consecutive de-spread values and are presented in [5][6][7]. Autocorrelation based algorithms employ the autocorrelation properties of the received signal and are discussed in [8][9]. The presence of a pilot symbol in a WCDMA system justifies the use of these data-aided estimation techniques.

An FFT spectrum of a signal of period T has a frequency resolution of $1/T$ Hz. In order to improve the performance of a conventional FFT frequency offset estimator in WCDMA systems, the use of quadratic interpolation to the peaks of the FFT spectral amplitudes was proposed in [2] and was shown to provide an improvement in performance. A similar but more accurate approach was used to estimate the frequency of sinusoidal components in music signals and the authors [10] studied the effect of window functions on the performance of the frequency estimator. This paper investigates the effect of overlapping partitions of the FFT window function on the performance of a quadratically interpolated FFT based frequency offset estimator in a WCDMA environment.

The rest of the paper is organised as follows. Section II introduces the problem of carrier frequency offset in a communications system. Section III discusses the conventional FFT based WCDMA frequency offset estimation algorithm. Section IV presents the proposed algorithm. The results are discussed in Section V and finally, a conclusion is drawn in Section VI.

II. CARRIER FREQUENCY OFFSET ESTIMATION

Carrier frequency offset arises when the oscillator at the receiver oscillates at a different frequency to that of the

Manuscript received May 17, 2008.

Seare H. Rezenom is with the School of Electrical, Electronic and Computer Engineering, University of KwaZulu-Natal, Durban, 4041 South Africa (e-mail: rezenoms@ukzn.ac.za).

Anthony D. Broadhurst was with the School of Electrical, Electronic and Computer Engineering, University of KwaZulu-Natal, Durban, 4041 South Africa.

transmitter [3]. This frequency error is seen as a frequency offset to the receiver. This section presents the generation of a carrier frequency offset and establishes the mathematical model used to formulate the problem of carrier frequency offset estimation.

In order to illustrate the generation of a carrier frequency offset, Fig. 1 shows a model used in a digital communication system using a crystal oscillator at the transmitter and receiver.

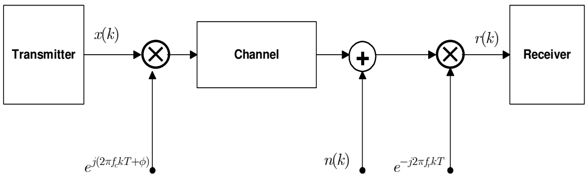


Fig. 1: Frequency offset modelling

The data to be transmitted $x(k)$ is up-converted using a crystal controlled oscillator with frequency f_c. The modulated signal is then propagated through a radio channel with an additive white Gaussian noise $n(k)$. At the receiver, the signal is down-converted using a crystal oscillator with frequency f_r and sampled at the chip period T to give $r(k)$.

Assuming a perfect channel, the received signal at sampling instant *k* can be expressed as

$$r(k) = \Big(x(k)\,e^{j(2\pi f_c kT + \phi)} + n(k)\Big)e^{-j2\pi f_r kT} \qquad (1)$$

where ϕ represents the phase offset at the transmitter. (1) can be further manipulated to give

$$r(k) = x(k)e^{j(2\pi (f_c - f_r)kT + \phi)} + n(k)e^{-j2\pi f_r kT} \qquad (2)$$

Multiplying both sides of (2) with the conjugate of $r(k)$ gives a parameter $y(k)$

$$y(k) = x^*(k)\,r(k) \qquad (3)$$

Therefore,

$$y(k) = x^*(k)\left\{x(k)e^{j(2\pi(f_c - f_r)kT + \phi)} + n(k)e^{-j2\pi f_r kT}\right\} \qquad (4)$$

Using the property of chipping sequences, $x^*(k).x(k) = 1$ and since the statistics of the second term in (4) is equivalent to the noise $n(k)$, (4) can be simplified further to

$$y(k) = e^{j(2\pi \Delta f\, kT + \phi)} + n(k) \qquad (5)$$

where $\Delta f = f_c - f_r$ is the carrier frequency offset. The challenge posed to the designer is to find some estimate that compensates the frequency error Δf introduced. Once a reliable estimate has been found, receiver structures (eg. the Phase Locked Loop) can be implemented to correct the frequency error.

III. CONVENTIONAL FFT BASED ALGORITHM

This section presents the conventional FFT-based WCDMA frequency offset estimator. The received signal $r(k)$ is de-spread using knowledge of the scrambling code sequence $c^*(k)$, where $(*)$ is a complex conjugate notation. The index k represents the scrambling code sequence number. The scrambling code sequence is obtained from the cell searching procedure which identifies the transmitted scrambling [2]. This is shown in Fig. 2 where $y(i)$ represents the complex (I and Q) de-spread signal.

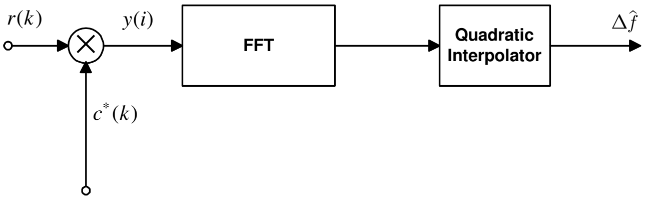


Fig. 2: Schematics of an FFT-based frequency offset estimator.

The received signal can be written as,

$$r(t) = \sqrt{p_k}\, d(t)\, c(t)\, e^{j(2\pi \Delta f t + \phi)} + \sqrt{p_n}\, n(t) \qquad (6)$$

where p_k represents the power of the complex scrambling code $c(t)$ used to spread the complex transmitted data signal $d(t)$ and $e^{j(2\pi\Delta f t+\phi)}$ is used to model the effect of the frequency offset Δf and phase ϕ. Additive White Gaussian Noise (AWGN) of zero mean and unity variance $n(t)$ is scaled with power p_n. The de-spread signal $y(i)$ is computed as,

$$y(i) = \sum_{k=0}^{S_F - 1} r(i \times S_F + k)\, c^*(i \times S_F + k), \qquad i = 0, 1, \ldots, P-1 \qquad (7)$$

where S_F is the de-spreading factor in chips and $P = (N_f \times 38400)/S_F$ is the total number of de-spread sequences in one frame and N_f represents the number of frames considered for estimation. Each WCDMA frame has a period of 10 ms and contains 38400 chips. Each frame is divided into 15 parts called 'slots'. A slot is made up of 2560 chips [1]. Using a chip duration T_c, (7) can be re-written as

$$y(i) = Y_0\, e^{j2\pi\Delta f \times S_F \times iT_c + \phi} \qquad i = \{0,1,...,P-1\} \qquad (8)$$

where Y_0 is the magnitude of the complex de-spread signal. The FFT coefficients of (8) are then computed with a rectangular window function with zero padding in order to use an FFT length of a power of 2. Quadratic interpolation is then applied to the peaks of the FFT. Fig. 3 illustrates the technique of finding the peak of the spectrum from three adjacent spectral lines. Point B (f_m, h_m) is the point of maximum FFT energy detected by the receiver, whereas points A (f_{m-1}, h_{m-1}) and C (f_{m+1}, h_{m+1}) are the spectral lines adjacent to it. The index m represents the position of the FFT peaks. The true frequency f is located at Point D.

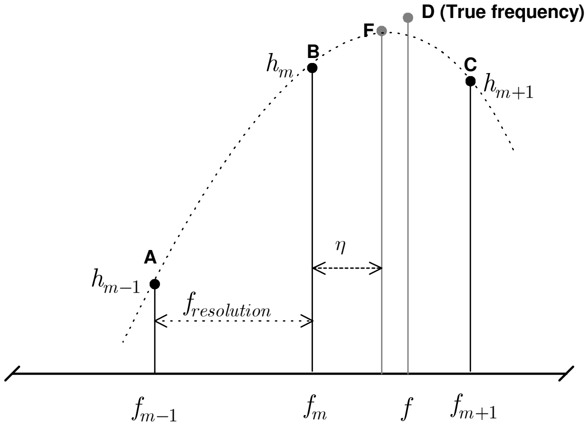


Fig. 3: Representation of the peaks of an FFT sequence.

A quadratic polynomial is then applied between the spectral peaks A, B and C to estimate the true peak which is located at point D. This operation gives point F and η is the bias arising from interpolation. The estimated carrier frequency offset error is the difference in frequency between points F and D, and is given by [2]

$$\Delta\hat{f} = f - f_m - \eta f_{resolution} \qquad (9)$$

where $\eta = \frac{1}{2}\left(\frac{h_{m-1} - h_{m+1}}{h_{m-1} - 2h_m + h_{m+1}}\right)$ and $f_{resolution}$ is the frequency resolution of the FFT.

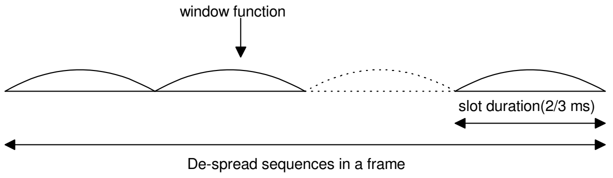
IV. PROPOSED METHOD

The FFT based estimator that was considered in the previous section calculates the Fourier transform of the de-spread sequences in a non-overlapping manner. Fig. 4(a) depicts this situation where window functions are applied to the de-spread sequences for the duration of every slot in a non-overlapping way. It is important to mention these windowed sequences were zero padded as explained in the previous section before the FFT was computed.

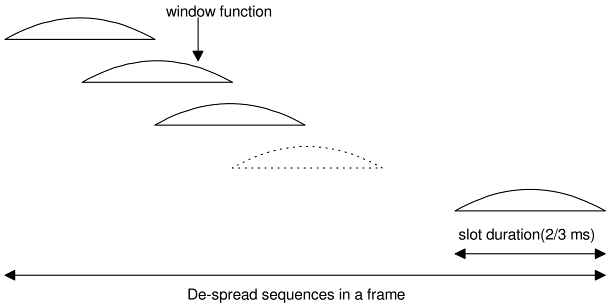
The proposed technique is inspired by the work done in [12] where the author reported the following observations about the non-overlapping technique of computing the Fourier Transform.

- If the window and the FFT are applied to non-overlapping partitions of the de-spread sequences, a significant part of the series is ignored due to the window exhibiting small values near the boundaries.
- If the transform is used to detect short duration tone-like signals, the non-overlapping method could miss the event if it occurred near the boundaries.

To avoid this loss of data, the transforms are usually applied to the overlapped partition sequences as shown in Fig. 4(b). In this paper, the case where the duration of overlap is 50% of the de-spread sequences per slot is considered. Instead of directly applying the window function on the de-spread sequences, the de-spread sequences are first aligned as shown in Fig. 4(b).



(a) Conventional method



(b) Proposed method

Fig. 4: Arrangement of de-spread sequences in a conventional and the proposed method.

V. RESULTS

The received data was generated as follows. A Gold code generator [1] was used to generate the scrambling code sequence needed to spread the data signal. Gold codes are chosen due to their good autocorrelation properties. Its length was then truncated to 38400 chips in order to satisfy the WCDMA frame length requirement of 10 ms. This data was then corrupted with a frequency offset at the start of the frame and AWGN added. The phase ϕ was assumed to be uniformly distributed in the interval 0 to 2π. At the receiver, the same Gold code was used to de-spread the received signal. A filtered white Gaussian noise model [11] is used to model the flat fading channel with a Doppler frequency of 9.26 Hz. Additional simulation parameters are presented in Table 1. The following procedures were then performed to estimate the frequency offset.

Step 1. De-spread signal using a spreading factor of 64.
Step 2. Collect 40 samples per slot.
Step 3. Apply a window function.
Step 4. Compute the 64 point FFT of the above sequence with 24 zeros to fill up the FFT window.
Step 5. Perform steps 1 – 4 on each slot in the available frames and find the average spectral amplitude at each bin frequency.
Step 6. Perform quadratic interpolation on the averaged FFT sequence.
Step 7. Apply (9) to estimate the frequency offset.

Table 1: Simulation parameters

Number of frames	1
FFT size (N)	64 point
De-spread values per slot (M)	40
Zero padding factor (Z_p)	1.6
De-spreading factor (S_F)	64
De-spread values per frame (P)	600
Slot duration (t_1)	2/3 ms
FFT window duration (t_2)	(64/40) x (2/3) ms
Resolution of FFT (1/t_2)	937.5 Hz

The performance of the proposed method is performed through simulations. Rectangular and Hanning window functions are used to compare the performance of the conventional and proposed algorithms. The algorithm presented in Figure 2 is used as a conventional algorithm. A 20 kHz frequency offset is assumed to be present in the received data. A flat fading channel with a Doppler shift of 9.26 Hz is considered. The fade duration is assumed to last one slot duration (2/3 ms). The error probability is taken as a measure of performance and is defined as the probability that the estimated frequency offset exceeds the true frequency by 200 Hz. Monte Carlo simulations with 1000 iterations were used for the simulations.

Fig. 5 shows the error probability of the proposed method in a flat fading channel. A frequency offset of 20 kHz is considered in the simulations. It is shown that the proposed method using the Rectangular and Hanning window functions performs better than the conventional FFT based algorithm in signal-to-noise ratios ranging from -40 dB up to -5 dB. For an error probability of 2 x 10^{-2}, an improvement close to 4 dB and 2 dB is recorded by the proposed method using Hanning and Rectangular windows, respectively, when compared to the conventional algorithm.

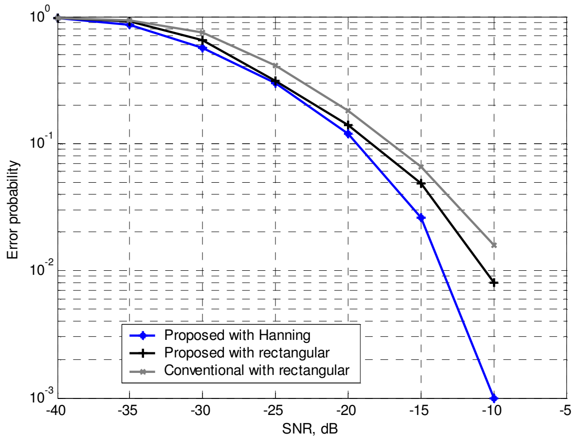


Fig. 5: Error probability of the proposed method

The main reason for this improvement is the fact that the proposed algorithm exploits the data-omissions caused by the windowing process. Another reason for the increase in performance can be attributed to the extra computations introduced by the overlapping sequences. For example for one frame estimation duration, 15 FFT operations are performed by the conventional algorithm. This compares with 29 FFT operations for the proposed algorithm. This brings about a refined frequency offset estimate in the averaging process of the Fourier transforms. Although it can be argued that this method increases the computational complexity of a WCDMA terminal, the good performance gain obtained would give a reason to compromise.

VI. CONCLUSION

This paper has presented a study of carrier frequency offset estimation in WCDMA systems. A simple technique based on partitioning the FFT window of the despread sequences is proposed and the results show an improvement when compared with conventional algorithms.

REFERENCES

[1] 3rd Generation Partnership Project, "Spreading and modulation (FDD)", 3GPP Tech. Spec., TS 25.213: V5.0.0 (2002-2003).

[2] Y.P.E Wang and T. Ottosson, "Cell search in W-CDMA" *IEEE Journal on Selected Areas in Comm.*, vol. 18, no. 8, pp. 1470-1482, Aug. 2000.

[3] Y. Jay Guo, "Advances in Mobile Radio Access Networks", Artech House Mobile Communication Series, 2004.

[4] T. Chulajata, H. Kwon, and K. Min, "Coherent slot detection under frequency offset for W-CDMA", *IEEE Vehicular Technology Conference*, pp. 1719-1723, May 2001.

[5] P. Rykaczewski, D. Pienkowski, R. Circa and B. Steinke, "Signal path optimization in software-defined radio systems", *IEEE Trans. Microwave Theory and Techniques*, vol. 53, no. 3, pp. 1056-1064, Mar. 2005.

[6] S.A. Tretter, “Estimating the frequency of a noisy sinusoid by linear regression,” *IEEE Trans. Inf. Theory*, vol. 31, no. 6, pp. 832-835, Nov. 1985.

[7] S. Kay, “A fast and accurate single frequency estimator,” *IEEE Trans. Acous., Speech, Signal Process*., vol. 37, no. 12, pp. 1987-1990, Dec. 1989.

[8] M.Morelli and M. Mengalli, “Carrier frequency estimation for transmission over selective channels,” *IEEE Trans. Comm.*. vol. 48, no. 9, pp. 1580- 1589, Sept. 2000.

[9] M.Morelli and M. Mengalli, “Feed forward frequency estimation for PSK: A tutorial review,” *Eur. Trans. Telecomm*., vol. 9, no. 2, pp. 103-108, Jan.-Apr. 1998.

[10] M. Abe and J. Smith, “CQIFFT: Correcting bias in a sinusoidal parameter estimator based on quadratic interpolation of FFT magnitude peaks”, Technical Report, STAN-M-117, Dept. of Music, Stanford University, Oct. 2004.

[11] D. Young and N. Beaulieu, “A quantitative evaluation of generation methods for correlated Rayleigh random variates,” *IEEE Global Telecomm. Conf.,* vol. 6, pp. 3332–3337, Nov. 1998.

[12] F. J. Harris, “On the use of windows for harmonic analysis with the discrete Fourier transform,” *Proceedings of the IEEE*, vol. 66, pp. 51-83, Jan. 1978.

Seare H. Rezenom (S’03) received the BSc (*honours*) degree in Electrical Engineering from the University of Zimbabwe, and the MScEng degree in Electronic Engineering from the University of KwaZulu-Natal, South Africa.

He is currently working towards the PhD degree in Electronic Engineering at the University of KwaZulu-Natal, South Africa. His current focus of research is on the synchronization techniques for multi-user MIMO-OFDM systems.

A.D. Broadhurst (M’77) received the BSc(Eng) degree in Electrical Engineering in 1965 and the PhD degree in 1970, both from the University of Cape Town.

He was a Professor of Communications in the School of Electrical, Electronic and Computer Engineering at the University of KwaZulu-Natal, Durban, South Africa. He is currently in retirement.