

Estimation of a Probability of Interference in a Cellular Communication Network using the Monte-Carlo Technique

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Abstract — A Monte-Carlo technique based methodology for the estimation of a probability of interference in a cellular communication network is presented. This probability is considered for the case of mobile to base (Up Link or Reverse channel) and base to mobile (Down Link or Forward channel) interferences. The effect of random user location in their respective cells is taken into account, and the impact of lognormal shadowing is also studied. The simulation results obtained, when compared with known results are found to be in agreement.

Index Terms — Cellular network, cochannel interference, Monte-Carlo method, probability of interference.

I. INTRODUCTION

The radio spectrum available for wireless data services and networks is extremely scarce, while demand for these services is growing at a rapid pace [1]. Spectral efficiency is therefore of primary concern in the design and planning of wireless communications networks. This efficiency is partly achieved by cellular networks which exploit the power falloff with distance of signal propagation to reuse the same frequency channel at spatially separated locations [2]. However, while frequency-reuse provides more efficient use of the limited available spectrum, it also introduces unavoidable cochannel interference [3]–[6], which ultimately determines the bit error rates (BER's) available to each mobile user.

The basic method for interference estimation in a communication network is by modeling and simulation, including computer experiment, performed on the basis of mathematical models of the network's functional components including: radio transmitters' emissions, radio receivers' receptivity, antenna feeder units, radio wave propagation, and different noise and interference mechanisms.

The principal unavoidable drawback in the deterministic methodology for interference estimation is the complexity of constructing mathematical models of the network's functional components due to the impossibility of analyzing a large number of interacting radio transmitters and radio receivers with a priori unknown parameters, and this is characteristic, for example, of the cellular communication networks. Since in a cellular communication network, the number and

locations of possible interfering sources are not known (e.g. when the interference is from land-mobile radio transmitters), the deterministic methodology of calculating reuse distance based on worst case network functioning condition is less adequate when compared with the probabilistic method. In cellular communication networks it is generally necessary to evaluate the aggregate effect of many independent signals on a radio receiver characterized not only by the presence of a set of random parameters and the random physical processes in the radio channel but also by differences in the functional algorithms of the networks. Therefore in order to fully access the interference potential of introducing a new network into an existing electromagnetic environment, the probability of interference due to multiple sources should be considered.

A methodology for calculating a probability of interference in a cellular communication network based on a "probabilistic" approach is presented in [7]. It defines a probability of interference that is dependent on the distance between co-channel base stations. Reuse distance are determined according to an acceptable level of interference probability, which is a function of specific system parameters, and also of the **off-channel rejection** (OCR) which is a measure of the capability of a receiver to reject interference. The above methodology is based on the calculation of the overlapping area between the coverage and the interfering area.

It is well known that signal propagation in a radio mobile environment is affected by three independent phenomena: deterministic path-loss variation with distance, random slow shadowing and random fast multipath fading. In the above mentioned methodology, the power levels are expressed only in terms of distances. Therefore performing statistical estimation of interference, while accounting for the set of random parameters of the signals and radio network, is practically impossible without the use of a special statistical method, one, of which is the Monte Carlo method.

The aim of this paper is to present a Monte-Carlo technique based approach for interference estimation which gives a probability of interference in a cellular communication network with the effect of random user location in their respective cells taken into account, and the impact of lognormal shadowing will also be studied.

II. NETWORK DESCRIPTION

Figure 2 shows the coverage areas (or cells) of two base stations (**BS₁** – wanted base station and **BS₂** – base station serving interferer mobile station) separated some D km from each other. Within both the wanted and interferer cells are mobiles stations (**MS₁** – wanted mobile station and **MS₂** –

interferer mobile station). The constant R_{cell} defines the radius of the base station's coverage areas. x_s is the distance between the wanted transmitter MS_s and the wanted receiver BS_1 (for mobile to base station interference) or between the wanted transmitter BS_1 and wanted receiver MS_s (for base to mobile station interference). $d_{MS \rightarrow BS}$ is the distance between the interfering mobile MS_i and the wanted base station BS_1 while $d_{BS \rightarrow MS}$ is the distance between the interfering base station BS_2 and the wanted mobile MS_s .

The mobile to base interference occurs when the wanted base station BS_1 is interfered with by an interfering mobile MS_i (Figure 2a). The base to mobile interference occurs when a wanted mobile MS_s is interfered with by an interfering base station BS_2 (Figure 2b).

The distance $d_{MS \rightarrow BS}$ from the interfering mobile MS_i to the wanted base station BS_1 (in the mobile to base station interference case) and $d_{BS \rightarrow MS}$ from the interfering base station to the wanted mobile MS_s (in the base station interference case) are calculated respectively by the expressions (1) and (2) as follows:

$$d_{MS \rightarrow BS} = \sqrt{(x_i \sin(\theta_i))^2 + (D + x_i \cos(\theta_i))^2}, \quad (1)$$

$$d_{BS \rightarrow MS} = \sqrt{x_s^2 + D^2 - 2x_s D \cos(\theta_i)}, \quad (2)$$

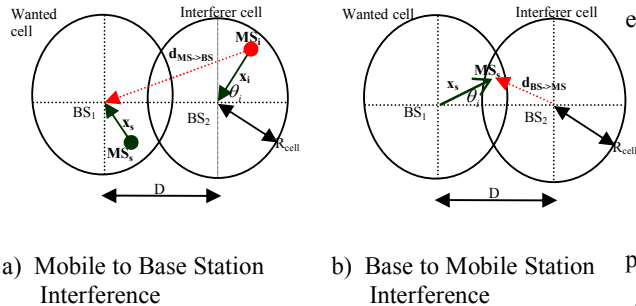


Figure 2: Wanted and Interfering Base Stations.

III. USERS' RANDOM LOCATION MODEL

For analytical convenience, we assume that the cell shape is approximated by a circle of radius R_{cell} . All the mobiles (desired and interfering users) are assumed to be mutually independent and uniformly distributed in their respective cells. Thus, the PDF of the mobiles polar coordinates (d and θ) relative to their BS's are

$$f(d) = \frac{2(d - R_{\min})}{(R_{\text{cell}} - R_{\min})^2}, \quad R_{\min} \leq d \leq R_{\text{cell}} \quad (3)$$

$$f(\theta) = \frac{1}{2\pi}, \quad 0 \leq \theta \leq 2\pi. \quad (4)$$

R_{\min} corresponds to the closest distance the mobile can be from the BS antenna, and is approximately equal to 20 m for microcellular systems and 80 m for macro-cellular systems.

IV. SIGNAL POWER LEVEL CALCULATION

It is well known that signal propagation in a radio mobile environment is affected by three independent phenomena: deterministic path-loss variation with distance, random slow shadowing and random fast multipath fading.

Path loss is due to the decay of the intensity of a propagating radio-wave. In both our analyses and simulations, we use the two-ray propagation model [8] to obtain the average received power as function of distance. According to this model, the average received signal power [W] is given by:

$$\frac{P_r}{P_t} = \left(\frac{Hh}{d^4} \right)^2, \quad (5)$$

where

P_r [W] — the received power level from a transmitter at a distance d [m] from the receiver,

P_t [W] — the transmitted power,

H [m] — base station antenna height

h [m] — mobile station antenna height

By considering the off-channel rejection - OCR (which is a measure of the capability of a receiver to reject interference) as a function of the frequency offset Δf between the transmitter and receiver centre frequencies, the above equation becomes

$$\frac{P_r}{P_t} = \left(\frac{Hh}{d^4} \right)^2 \times \frac{1}{OCR(\Delta f)}, \quad (6)$$

The value of OCR is computed in [9].

Therefore in logarithmic units, the received signal power P_r [dBm] can be estimated as follows:

$$P_r = 20 \log(Hh) + P_t + G_t + G_r - 40 \log(d) - OCR(\Delta f), \quad (7)$$

where:

P_t — transmitted signal power [dBm]

G_t — transmitter antenna gain [dB]

G_r — receiver antenna gain [dB]

$OCR(\Delta f)$ — Frequency offset [dB] between the transmitter and receiver centre frequencies, which is computed in [9]. If $\Delta f = 0$ (i.e. no frequency offset between interfering networks), then $OCR(\Delta f) = 0 \text{ dB}$.

In a mobile communication system, the two-ray propagation model used above and many other propagation models like the Hata model, the ITU-R P.529-3 model and the ERC Report 68 model estimates the mean value of the signal power level to reasonable degree of accuracy but does not give the statistics caused by the effects of shadowing of the line-of-sight path from terrain, buildings, and trees in other words known as slow fading, and multipath fading due to the combination of randomly delayed reflected, scattered and diffracted signal components in other words known as fast (relative to the shadowing effect) fading.

The shadowing is generally modelled as lognormal distributed [10]. The probability density function (PDF) of the

slowly varying received signal power is thus given by the lognormal expression:

$$f(P_s) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(P_s - P_r)^2}{2\sigma^2}\right), \quad (8)$$

where:

σ — Standard deviation [dB]

P_r — Mean value of signal power [dBm]

P_s — Instantaneous value of the received signal power [dBm].

According to many sources the multipath fading environment is generally modelled as Rayleigh, Rician or Nakagami - distribution.

V. INTERFERENCE CRITERIA

To simplify the analyses, the following assumption has been made in the interference model. We consider interference-limited systems in which the thermal noise power is negligible relative to the cochannel interference power [11]. Thus, the ratio of signal power P_i [W] to noise plus interference power P_i [W] reduces to the signal-to-interference power ratio P_s/P_i (SIR).

Harmful interference is assumed to occur if the SIR is less than a certain protection ratio ε i.e.

$$\frac{P_s}{P_i} < \varepsilon, \quad (9)$$

When expressed in logarithmic unit, the condition for harmful interference is that interference power level P_i [dBm] must be higher than the wanted signal power level P_s [dBm] by a certain protection ratio ε [dB] as expressed by (10):

$$P_i - P_s > -\varepsilon. \quad (10)$$

VI. MONTE CARLO METHOD

The Monte Carlo method is a well known method for the estimation of the statistical characteristics of a random variable $y = f(x_1, x_2, x_3, \dots, x_N)$, where $x_1, x_2, x_3, \dots, x_N$ are random variables, in cases where it is analytically difficult to obtain the integral probability distribution function $F(y)$ of y , but the integral probability distribution functions $F_1(x_1)$, $F_2(x_2)$, $F_3(x_3)$, ..., $F_N(x_N)$ are analytically defined [13].

The main idea of the Monte Carlo method is as follows: Suppose that the distribution of a random variable x is known (**Figure 1**). If we randomly take any number $\xi_i \in (0,1)$ from the ordinate axis and find the corresponding x_i for which $F(x_i) = \xi_i$, then the random number x_i will have the integral probability

distribution function $F(x_i)$, i.e. we have the functional transformation $x_i = F^{-1}(\xi_i)$.

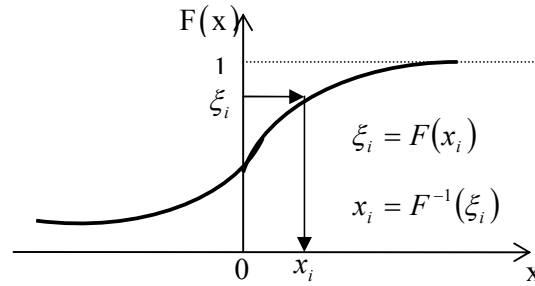


Figure 1: Sampling from a Probability Distribution

Therefore modelling and simulation by the Monte Carlo method is a process of obtaining with the aid of a computer, n realizations of a series of randomly sampled values of the variable x_1, x_2, x_3, \dots from given distributions.

The Monte - Carlo method enables us to simulate the distribution of instantaneous values of the wanted signal and interference power levels at the receivers input accounting for as many as possible random factors of known distributions, without having to represent them by their analytical expressions.

VII. MONTE-CARLO SIMULATION AND RESULTS

Since the signal power of both the wanted and interfering mobiles experience fluctuations due to multipath fading, shadowing, and the random location of users in their respective cells, the signal-to-interference power ratio is also a random variable which depends on the distribution of P_s and P_i . Therefore to fully access the effect of the interaction between the wanted signal and interference power levels at the receiver input, these levels must be described by suitable distribution functions.

The Monte-Carlo method is base on the generation and processing of multiple random instances of the network, each of which is characterised by the instantaneous values of mobile users locations for the wanted and interferer mobile obtained by (3) and (4) and the shadowing effect on the wanted and interference power levels are obtained according to (8).

The distribution of the instantaneous values of the wanted signal and interference power levels at the receiver's input and hence a probability of interference can be estimated using Monte-Carlo technique as follows:

1. We start by generating 3 independent random numbers, two of them d_s, x_i corresponding to the random distances of the wanted mobile user \mathbf{MS}_s from the wanted base station \mathbf{BS}_s and of the interfering mobile user \mathbf{MS}_i from its serving base station (\mathbf{BS}_i) respectively are uniformly distributed within the cells of coverage radius R_{cell} according to (3), the third number θ_i is uniformly distributed between 0 and 2π according to (4).
2. Using (1) we estimate the random distance $d_{\mathbf{MS}_s \rightarrow \mathbf{BS}_s}$ of the interfering mobile \mathbf{MS}_i from the wanted base station \mathbf{BS}_s for the mobile to base station interference and (2) for the random distance $d_{\mathbf{BS}_s \rightarrow \mathbf{MS}_s}$ of the interfering base station \mathbf{BS}_i from the wanted mobile \mathbf{MS}_s .

- By using expression (7), we estimate the power level of the signals from the wanted mobile station MS_s and interfering mobile station MS_i at the input of the wanted base station BS_s receiver for the mobile to base station interference and from the wanted BS_s and interfering BS_i base station at the input of the wanted mobile station MS_s for the base station to mobile interference. These power levels are then used as the mean values for the generation of the instantaneous signal and interference power levels at the receiver input.
 - Conditions (10) is verified and the outcome noted
 - Repeat step 1 – 4 a large number (N) times.
- The probability of interference P_{prob} is then estimated as the ratio:

$$P_{prob} = \frac{K}{N}$$

where:

K — Number of times for which the condition is fulfilled.

Based on the above presented methodology, a probability of interference in a cellular communication network for the mobile to base and base to mobile interference is calculated for various values of OCR and re-use distance D . The distribution of mobile users inside both the victim and interfering coverage areas is assumed to be uniform. The MATHCAD simulated probability curves for mobile to base station and base to mobile station, for the case where the wanted signal and interference lognormal shadowing are (a) not accounted for and (b) accounted for, are plotted in Figure 3 and Figure 4 respectively for $N = 50,000$ samples. The network parameters [7] used in the simulation are shown in the Table 1 below.

Table 1: Parameters used in the Simulation

Parameter	Value
Base station (Mobile station) power, dBm	43(30)
Base station (Mobile station) antenna height, m	50(1.5)
Base station (Mobile station) antenna gain, dB	5(0)
Protection ratio, dB	18
Cell radius, km	32
Standard deviation, dB	8

VIII. CONCLUSION

The curves of a probability of interference vs. the reuse distance in a cellular communication network obtained by simulation according to the Monte–Carlo based methodology introduced here is in agreement with those obtained in [7]. The effect of random user location in their respective cells is taken into account, and the impact of lognormal shadowing is also studied, but in general this methodology when compared with those in [7], permits us to account for any number of random parameters affecting the network’s functioning without any further difficulties. As expected, the probability of interference with lognormal shadowing accounted for decreases with

increase in reuse distance but for certain values of OCR, the probability will actually increase and then decrease as

Probability of interference for mobile to base station (a) without lognormal shadowing (b) with lognormal shadowing

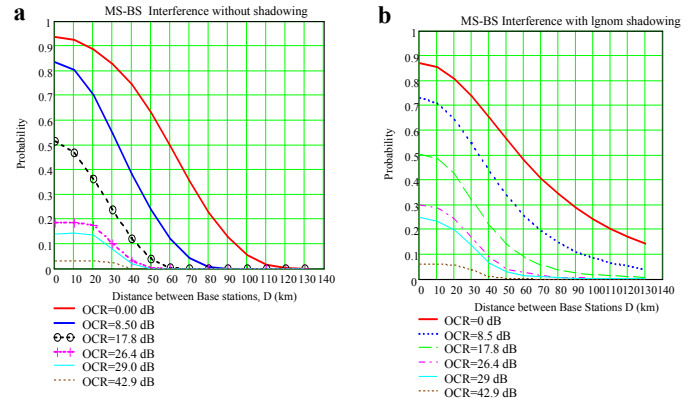


Figure 3: Probability of Interference for Mobile to Base Station

Probability of interference for base to mobile station (a) without lognormal shadowing (b) with lognormal shadowing

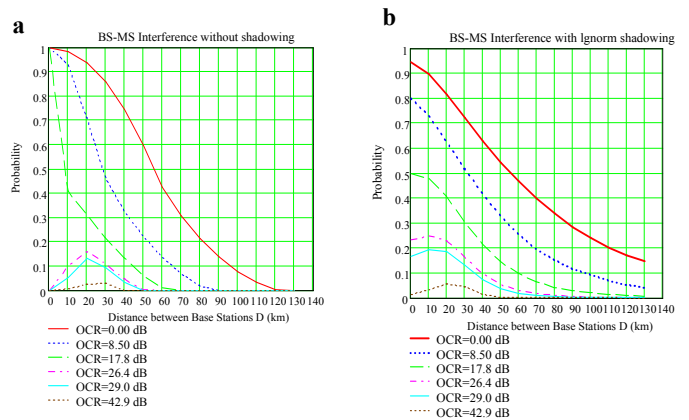


Figure 4: Probability of Interference for Base to Mobile Station

the reused distance increases. This is the case, for example when OCR is more than or equal to 26.4.

These curves can be used to determine the minimum reuse distance for an acceptable probability of interference hence increasing the spectral efficiency of the cellular communication network. For example, given an OCR value of 8.5 dB and an acceptable probability of interference of 0.05, the curves give us a reused distance of 72 km for the base to mobile interference and 67 km for the mobile to base interference. Hence, 67 km should be chosen as it guarantees minimum requirements for both type of interference.

These results are suitable for practical use by operators and planners of cellular communication networks during the network planning and optimization process.

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