Channel Capacity Performance of Transmit Antenna Selective MIMO System in Weibull Fading

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Abstract—Multiple-input multiple-output (MIMO) systems play a great role in improving communication in wireless systems. But an increase in the number of antennas used in a communication system results in increased system complexity. To address this drawback associated with MIMO systems we propose the use of a transmit antenna selective MIMO system. In this the best performing transmit antenna is selected from all the available antennas for communication. The signal-to-noise ratio (SNR) performance of the proposed system is studied as the number of antennas used in the system is increased. In studying the SNR performance, maximal-ratio-combining (MRC) is used as the optimal combining technique at the receiver end. Average channel performance is also investigated as we vary the number of antennas used. The Weibull fading channel is considered in studying the performance of the transmit antenna selective MIMO system. We further illustrate the effect of the Weibull fading parameter on the average channel capacity performance.

Index Terms—Average channel capacity, maximal-ratio-combing, transmit antenna selection, Weibull fading.

I. INTRODUCTION

The increasing demand for high-rate data and/or better quality services has created a great need for improved bandwidth efficiency in wireless communication systems. Several diversity techniques have been proposed to try to improve the performance of wireless communication system. These techniques help in combating the effects associated with fading and multipath propagation. They include amongst others time diversity, frequency diversity and antenna diversity. Out of all these, antenna diversity has been observed to be bandwidth efficient and this has led to the development of multiple-input multiple output (MIMO) systems.

MIMO systems are able to provide improved data rates or better quality signals through the use of multiple transmit and multiple receive antennas [1]. This improvement in performance is associated with encoding the modulated signals using space-time coding techniques and then transmitting the encoded signals using different antennas [2]. To improve on data rates the signals are encoded using Bell Laboratories Layered Space-Time (BLAST). Whilst to improve on the quality, the signals are encoded using space-time trellis (STT) coding or space-time block (STB) coding. STB coding is considered in this study due to the fact that it has less decoder complexity compared to STT coding. This is because the simple maximum likelihood technique can be used to decode signals which have been encoded using STB coding. The performance of STB codes was first studied by Alamouti who showed that diversity gain could be achieved by using two transmit antennas. Diversity gain implies that it is highly unlikely for all the signals to simultaneously fall in the deep fade region. His study was later extended to any number of transmit antennas by Tarok *et al*. [3]

One drawback with MIMO systems is the increase in complexity associated with an increase in the number of antennas used in the system [4], [5]. To help solve this problem, antenna selection was proposed [6], [7]. The idea with antenna selection is to select the best performing antenna(s) and using this instead of all the available antennas for communication. In this way the number of radio frequency chains to be decoded at the receiver is reduced. Antenna selection is applicable at the transmitter and/or receiver end. In this correspondence transmit antenna selection is considered. With transmit antenna selection the single best performing transmit antenna is selected. The selection is such that the antenna that maximizes the signal's SNR is chosen.

The signals are then combined at the receiver end using several combining techniques, namely, maximal-ratio-combining (MRC), equal-gain-combining (EGC) and selection combing (SC). Of all these combining techniques, maximal-ratio-combining is the best performing and it's the one we will consider when studying the performance of the transmit antenna selective MIMO system [8].

In this paper we consider the Weibull fading channel in investigating the SNR performance with an increase in the number of antennas in the system and when MRC has been used. We also study the average channel capacity performance of the transmit antenna selective MIMO system.

The rest of the paper is organized as follows. The system model is presented in section II. In section III the Weibull fading channel is presented. Sections IV and V provide the SNR and average channel capacity analysis for the transmit antenna selective MIMO system respectively. In section VI, simulation results are presented. Finally, the paper is concluded in section VII.

II. SYSTEM MODEL

We consider an $(n_T, 1; n_R)$, Fig. 1 wireless link in a flat Weibull fading environment where n_T is the number transmit antennas, n_R the number of receive antennas and 1 is the selected single transmit antenna. The fading coefficients $h_{ij}, 1 \le i \le n_T, 1 \le j \le n_R$ are modeled as independent samples of complex Gaussian random variables with a zero mean and the variance of 0.5 per dimension. It is assumed that the channel state information is perfectly known at the receiver end and partially known at the transmitter end through a feedback channel. At any time, only one of the total n_T antennas is chosen and activated for further transmission.

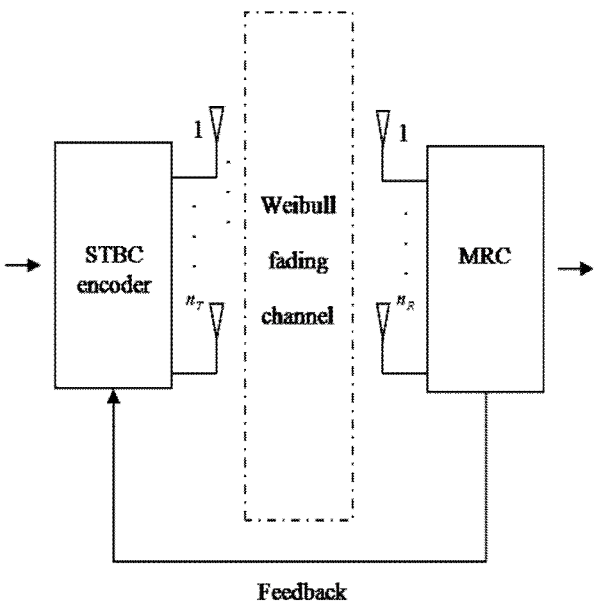

Fig. 1. Transmit antenna selective MIMO system model.

The selection is such that only the single transmit antenna with the highest signal, SNR is selected from all the available transmit antennas. The single selected transmit antenna, denoted as Tx, is determined by [9], [10]

$$Tx = \arg\max_{1 \le j \le n_T} \left\{ Tx_i = \sum_{i=1}^{n_R} h_{ij}^2 \right\} \tag{1}$$

After the selection, all the transmit power is then concentrated on this particular antenna. Transmission will continue until the BER falls to a set threshold, after which all the transmit antennas will be allowed to start the transmission process and then the selection takes place again. Having selected the best transmit antenna, the signals at the receiver end are then combined using MRC.

III. WEIBULL FADING

In a MIMO system with flat fading wireless channel, the received signal can be modelled as follows:

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n} \tag{2}$$

where x is an $(n_T \times 1)$ transmitted signal vector, with n_T being the number of transmitters. y is an $(n_R \times 1)$ received signal vector, with n_R being the number of receivers. H is the $(n_R \times n_T)$ channel matrix and n is an $(n_R \times 1)$ additive white Gaussian noise vector. The channel matrix H is given as

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n_T} \\ h_{21} & h_{22} & \cdots & h_{2n_T} \\ \vdots & \vdots & \vdots & \vdots \\ h_{n_R 1} & \cdots & \cdots & h_{n_R n_T} \end{bmatrix} \tag{3}$$

where h_{ij} describes the channel gain between the i_{th} receiving antenna and the j_{th} transmitting antenna.
For the Weibull fading model, the complex envelope h_{ij} can be written as a function of the Gaussian in-phase X_{ij} and quadrature Y_{ij} elements of the multipath components [11]

$$h_{ij} = (X_{ij} + jY_{ij})^{2/\beta_{ij}} \tag{4}$$

where $j = \sqrt{-1}$ and β_{ij} is the fading parameter.

Let Z_{ij} be the magnitude of h_{ij}, i.e. $Z_{ij} = |h_{ij}|$, the Weibull fading model, Z_{ij} can be expressed as a power transformation of a Rayleigh distributed random variable (RV), $R_{ij} = |X_{ij} + jY_{ij}|$ as [12]

$$Z_{ij} = R_{ij}^{2/\beta_{ij}} \tag{5}$$

From (5), the PDF of Z_{ij} can be given as [13]

$$fz_{ij}(r) = \frac{\beta_{ij}}{\Omega_{ij}} r^{\beta_{ij}-1} \exp\left(-\frac{r^{\beta_{ij}}}{\Omega_{ij}}\right) \tag{6}$$

with $\Omega_{ij} = E(Z_{ij}^{\beta_{ij}})$. β_{ij} is the fading parameter expressing the fading severity (β_{ij}>0) and Ω_{ij} is the average fading power. As β_{ij} increases, the effect of fading decreases, while for the special case of $\beta_{ij} = 2$, the Weibull PDF of Z_{ij} reduces to the Rayleigh PDF. Whilst for $\beta_{ij} = 1$ the Weibull PDF of Z_{ij} reduces to the well known negative exponential PDF.

The corresponding CDF of Z_{ij} can be expressed as

$$Fz_{ij}(r) = 1 - \exp\left(-\frac{r^{\beta_{ij}}}{\Omega_{ij}}\right) \quad (7)$$

IV. Signal-To-Noise Ratio

In Weibull fading the instantaneous signal-to-noise ratio at the input of the receiver is given by [14]

$$\gamma_{ij} = Z_{ij}^2 \frac{E_s}{N_o} \quad (8)$$

and the average SNR is then given as [14]

$$\bar{\gamma}_{ij} = E(Z_{ij}^2)\frac{E_s}{N_o} = \Omega^{2/\beta_{ij}} \Gamma\left(1+\frac{2}{\beta_{ij}}\right)\frac{E_s}{N_o} \quad (9)$$

Based on an interesting property of the Weibull distribution, that the n_{th} power of a Weibull distributed random variable with parameters $(\beta_{ij}, \Omega_{ij})$ is another Weibull distributed random variable with parameters $(\beta_{ij}/n, \Omega_{ij})$ [14]. From the above mentioned property it can then be concluded that γ_{ij} is also a Weibull random variable with parameters $(\beta_{ij}/2, (a_{ij}\bar{\gamma}_{ij})^{\beta_{ij}/2})$ where $a_{ij} = 1/\Gamma(1+2/\beta_{ij})$. The PDF of γ_{ij} can then be derived from (6) by replacing β_{ij} with $\beta_{ij}/2$ and Ω_{ij} with $(a_{ij}\bar{\gamma}_{ij})^{\beta_{ij}/2}$ as [13]

$$p_{\gamma_{ij}}(\gamma) = \frac{\beta_{ij}}{2\left(a\bar{\gamma}_{ij}\right)^{\beta_{ij}/2}}\left(\gamma_{ij}\right)^{(\beta_{ij}/2)-1} \exp\left[-\left(\frac{\gamma_{ij}}{a\bar{\gamma}_{ij}}\right)^{\beta_{ij}/2}\right] \quad (10)$$

When MRC is applied at the receiver end the instantaneous SNR measured at the output of the two receivers is given as

$$\gamma_{MRC} = \gamma_{1j} + \gamma_{2j} \quad (11)$$

where γ_{1j} denotes the instantaneous SNR between the first receive antenna and the selected *jth* transmit antenna and γ_{2j} denotes the instantaneous SNR between the second receive antenna and the selected *jth* transmit antenna. The average SNR is written as

$$\bar{\gamma}_{MRC} = \bar{\gamma}_{1j} + \bar{\gamma}_{2j} \quad (12)$$

where $\bar{\gamma}_{1j}$ denotes the average SNR between the first receive antenna and the selected *jth* transmit antenna and $\bar{\gamma}_{2j}$ denotes the average SNR between the second receive antenna and the selected *jth* transmit antenna.

This average SNR will then be used in studying the average channel capacity performance of the transmit antenna selective MIMO system under the Weibull fading channel.

V. Average Channel Capacity

The standard formula for the Shannon capacity is [15]

$$C = BW \log_2(1+\gamma_{ij}) \quad (13)$$

The instantaneous capacity of a MIMO system with no CSI at the transmitter is given by [16] – [18]

$$C = BW \log_2\left[\det\left(\mathbf{I}_{n_R} + \frac{\rho}{n_T}\mathbf{HH}^{\dagger}\right)\right] \quad (14)$$

where $\rho = E_s/N_o$ is the average SNR per receiving antenna, with E_s as the average power at the output of each of the receiving antennas and N_o the corresponding noise power. $\mathbf{I}_{n_R}$ is the $(n_R \times n_R)$ identity matrix and † denotes the conjugate and transposition and BW is the bandwidth.

We then consider the average channel capacity, which in Shannon's sense is given as [13], [19]

$$\bar{C} = BW\int_0^{\infty} \log_2(1+\gamma_{ij})\, p_{\gamma_{ij}}(\gamma)\, d\gamma \quad (15)$$

where $p_{\gamma_{ij}}(\gamma)$ is the PDF of the signal-to-noise ratio and is given by (10) , and BW the bandwidth. Substituting (10) into (15), for Weibull fading the average channel capacity can be written as

$$\bar{C} = \frac{BW\beta}{2\left(a\bar{\gamma}_{ij}\right)^{\beta/2}\ln(2)}\int_0^{\infty} \gamma_{ij}^{(\beta_{ij}/2)-1} \ln(1+\gamma_{ij}) \times \exp\left[-\left(a\bar{\gamma}_{ij}\right)^{-(\beta_{ij}/2)} \gamma^{\beta_{ij}/2}\right] d\gamma \quad (16)$$

The above integral can be evaluated in closed-form by expressing the logarithmic and exponential integrands as Meijer's G-functions as [13], [20]

$$\ln(1+\gamma_{ij}) = G_{2,2}^{1,2}\left[\gamma_{ij}\,\middle|_{1,0}^{1,1}\right]$$

and

$$\exp\left\{-\left[\gamma_{ij}/\left(a\bar{\gamma}_{ij}\right)\right]^{\beta_{ij}/2}\right\} = G_{0,1}^{1,0}\left[\left[\gamma_{ij}/\left(a\bar{\gamma}_{ij}\right)\right]^{\beta_{ij}/2}\,\middle|_{0}^{-}\right].$$

The average channel capacity can then be written in closed-form as

$$\bar{C} = \frac{\beta\left(a\bar{\gamma}_{ij}\right)^{-(\beta_{ij}/2)}}{2\ln(2)} \frac{BW\sqrt{k}\,l^{-1}}{\left(\sqrt{2\pi}\right)^{k+2l-3}} \times G^{k+2l,l}_{2l,k+2l}\left[\frac{\left(a\bar{\gamma}_{ij}\right)^{-\beta_{ij}k/2}}{k^k}\,\middle|\, \frac{I\left(l,-\frac{\beta_{ij}}{2}\right), I\left(l,1-\frac{\beta_{ij}}{2}\right)}{I(k,0), I\left(l,-\frac{\beta_{ij}}{2}\right), I\left(l,-\frac{\beta_{ij}}{2}\right)}\right] \qquad (17)$$

where $I(n,\xi) \cong \frac{\xi}{n}, \left(\frac{\xi+1}{n}\right), \ldots, \left(\frac{\xi+n-1}{n}\right)$ with ξ as an arbitrary real value and n as a positive integer. Furthermore, $\frac{k}{l} = \frac{\beta}{2}$ where k and l are positive integers, depending on the value of β, e.g. for $\beta = 1.5$ we choose $k = 4$ and $l = 3$ and for $\beta = 2$ we choose $k = 2$ and $l = 2$.

VI. Simulation Results

In this section we present the SNR and average channel capacity performance of the transmit antenna selective MIMO system. The effect on system performance when changing the number of transmit antennas and also when changing the fading parameter, β is illustrated. To study these performance parameters, simulations were done using MATLAB and Mathematica. The signals were assumed to be modulated using binary-phase-shift keying (BPSK) and encoded using STB coding. The antennas were assumed to be sufficiently spaced so as to avoid any interference between neighbouring antennas. The channel state information (CSI) was assumed to be available at the receiver end and partially available at the transmitter end through the feedback channel. The total number of transmit antennas used ranged between two and four whilst the receive antennas were kept at two. The transmit antenna selective MIMO system is presented as $(n_T, 1; n_R)$, where only one of the total n_T transmit antennas is chosen and activated for further transmission and n_R is the number of receive antennas.

Fig. 2 is a plot of number of antennas used in a wireless communication MIMO system against the signal-to-noise ratio for three combining techniques, namely, maximal-ratio-combining (MRC), equal-gain-combining (EGC) and selective combining (SC). The MRC technique is observed to outperform the other two combining techniques. For this reason it was selected as the optimum combining technique for studying the performance of the transmit antenna MIMO system.

Literature tells us that the SNR in a MIMO system can be improved by increasing the number of antennas used in the system. Fig. 2 illustrates the SNR improvement with increase in the number of antennas used. This improvement in SNR leads to enhanced performance as the signals can be decoded with reduced probability of error at the receiver end. Thus MIMO systems play a great role in mitigating the effects of fading in wireless communication systems.

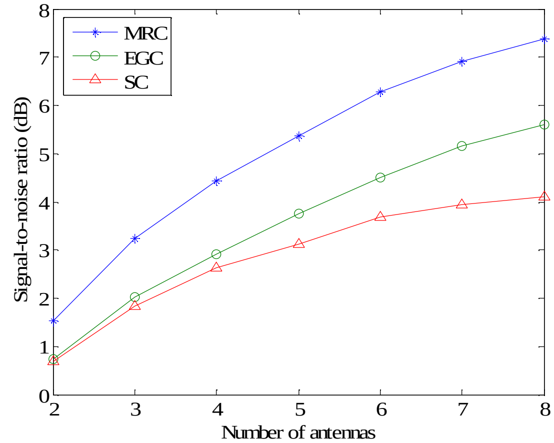

Fig. 2. SNR performance for MRC, EGC and SC with increasing number of antennas.

Using (17), Fig. 3 and Fig. 4 were plotted. These are plots of the average SNR against the normalized average channel capacity for different transmit antenna selective MIMO systems with different values of fading parameter β. The SISO system is included just for comparison purposes. As the total number of transmit antennas is increased in the transmit antenna selective MIMO system the channel capacity performance also improves, i.e. (4,1;2) > (3,1;2) > (2,1;2). This shows that MIMO systems can be used to improve the channel capacity without bandwidth expansion thus leading to improvement in communication.

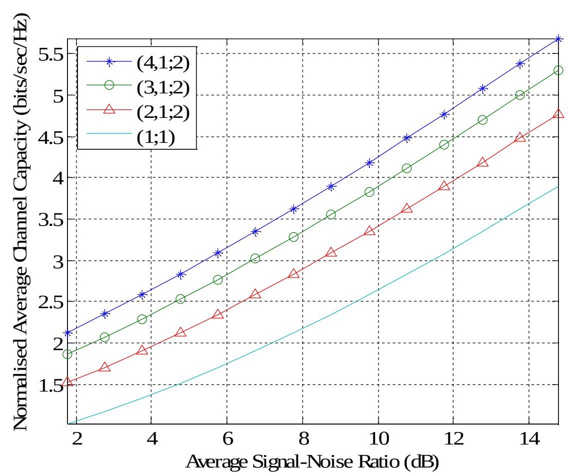

Fig. 3. Average SNR as a function of the normalized average channel capacity for $\beta = 1.5$.

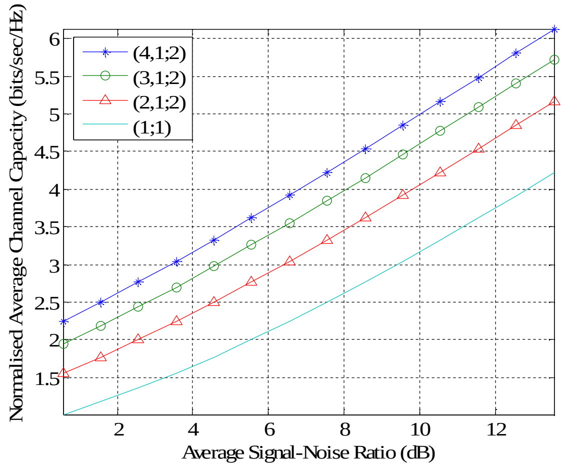

Fig. 4. Average SNR as a function of the normalized average channel capacity for $\beta = 3$.

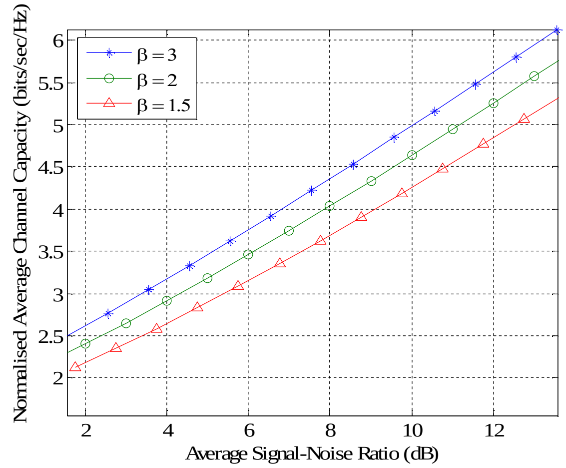

Fig. 6. Average SNR as a function of the normalized average channel capacity for the (4,1;2) transmit antenna selective MIMO system, with $\beta = 1.5$, 2 and 3.

Fig. 5 and Fig. 6 are plots of average SNR against the normalized average channel. In each graph the same transmit antenna selective MIMO system is considered but for different values of fading parameter. As the value of β increases the average channel capacity is improved. An increase in β results in a decrease in the effects of fading, thus improvement in the performance of a wireless communication system.

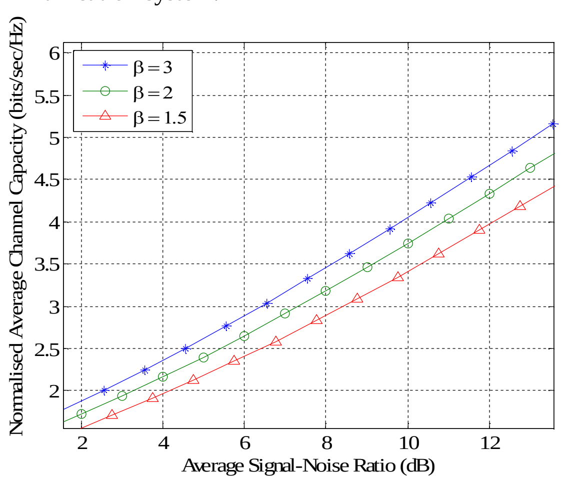

Fig. 5. Average SNR as a function of the normalized average channel capacity for the (2,1;2) transmit antenna selective MIMO system, with $\beta = 1.5$, 2 and 3.

VII. Conclusion

The performance of the transmit antenna selective MIMO was investigated. The SNR improvement with MRC and increasing the number of antennas in the system was illustrated. It was shown that using multiple antennas in wireless system improves the SNR. Improvement in SNR implies signals can be received at the receiver end with minimum errors thus allowing for easier decoding. The average channel capacity performance was also demonstrated for various system configurations. The proposed transmit antenna selective MIMO system was shown to outperform the SISO system. Channel capacity performance was also demonstrated to improve by increasing the fading parameter. This is because an increase in the fading parameter results to a decrease in the effects of fading on the wireless communication system.

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