

A Low Complexity Recurrent Neural Network MLSE Equaliser: Applications and Results

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Abstract—In this paper a soft output low complexity neural network based iterative Maximum Likelihood Sequence Estimation (MLSE) equaliser, based on earlier work by the authors, is evaluated for different communication systems. The equaliser is evaluated for underwater acoustic- and Code Division Multiple Access (CDMA) communication systems, and a method for exploiting the low complexity optimisation ability of the equaliser is also demonstrated using multiple transmit antennas. The equaliser is evaluated in environments where the channel delay spread is very long compared to the symbol period. Long delay spreads imply long channel impulse response (CIR) lengths, or channel memory lengths, making the process of optimal equalisation very difficult, as the computational complexity of optimal equalisers that are currently being used is exponentially related to the CIR length. It was shown in earlier work by the authors that the iterative MLSE equaliser is able to equalise signals in systems with hundreds of interfering symbols, since its computational complexity is approximately independent of the CIR length. The results presented in this paper emphasises the ability of the proposed equaliser to outperform suboptimal equalisers that are currently being used in systems with extremely long CIR length, while doing so at very low computational cost.

I. INTRODUCTION

Multipath propagation in wireless communication systems is a challenge that has enjoyed much attention over the last few decades. This phenomenon, caused by the arrival of multiple delayed copies of the transmitted signal at the receiver, results in intersymbol interference (ISI). Equalisation is necessary to mitigate the effect of ISI, in order to estimate the original transmitted symbols with maximum confidence.

In the early 1970's, Forney proposed an optimal equaliser [1] based on the Viterbi algorithm (VA) [2], proposed a few years before for the optimal decoding of convolutional codes, able to optimally estimate the most likely sequence of transmitted symbols. Shortly afterwards, the BCJR algorithm [3], named after its inventors, also known as the maximum a posteriori (MAP) algorithm, was proposed, able to produce optimal estimates for the transmitted symbols. The development of an optimal MLSE equaliser was an extraordinary achievement, as it enabled wireless communication system designers to design receivers that can optimally detect a sequence of transmitted symbols, corrupted by ISI, for the first time.

Although these algorithms estimate the transmitted information with maximum confidence, their computational complexities are prohibitive. The computational complexities of these optimal algorithms are linear in the data block length

and exponential in the CIR length, rendering them practically infeasible in communication systems with moderate to large bandwidth. For this reason, communication system designers are forced to use suboptimal equalisation algorithms to alleviate the computational strain of optimal equalisation algorithms.

One technique often used is to filter the received symbols with a Minimum Mean Square Error (MMSE) filter in order to concentrate most of the signal energy in the first few taps of the channel impulse response (CIR), so that the VA or MAP algorithms can be used for equalisation by using only the leading taps. A linear MMSE equaliser is also often used to determine a set of filter coefficients to render the error terms orthogonal to the estimated transmitted symbols [4], before filtering the ISI corrupted received symbols to produce the estimated sequence of symbols. Although these techniques provide some performance improvement, they all cause noise enhancement, leading to suboptimal equalisation.

Other techniques from the class of decision feedback equalisers (DFE) are also proposed, where the received symbols are filtered with an MMSE prefilter in order to produce a non-minimum phase CIR, where after symbol-by-symbol decision are made and fed back to be used in subsequent decision stages [4]. Delayed decision feedback DDFE is proposed in [5] where the first few taps are equalised using a reduced state trellis, where after the ISI caused by the remaining CIR taps is alleviated via conventional DFE.

In Code Division Multiple Access (CDMA) systems, there may be tens to hundreds of memory elements, rendering even the best opportunistic equalisers infeasible. The approach taken to equalise signals in a CDMA system is to use a RAKE demodulator. The RAKE demodulator uses fingers to select the highest energy taps in the CIR to coherently recombine the energy that was spread by the channel [4]. The processing gain in a CDMA system compensates for the losses incurred during energy recombination, enabling transmission of multiple data streams on one frequency. This technique is by no means optimal.

Signals in underwater communication systems are also subjected to ISI caused by potentially hundreds of interfering symbols [6]. This is due to the nature of the underwater acoustic channel (UAC), where high Doppler frequencies are also prevalent, causing rapidly varying channels, further complicating equalisation [7]. Here DFE approaches are common, and use of orthogonal frequency division multiplexing (OFDM) [8] has also been proposed to alleviate the detrimental effect of hundreds of interfering symbols. However, due to long CIRs and high Doppler spreads, neither of these of these techniques achieve desirable system performance.

In this paper the iterative MLSE equaliser developed in [9], and expanded upon in [10] and [11], will be evaluated against conventional equalisation schemes used in underwater-

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and CDMA communication systems. A multiple transmit antenna scheme is also proposed to utilise the low complexity equalisation ability of the proposed equaliser, using spatial diversity to enhance system performance. It was shown in [9], [10], and [11], that the computational complexity of the equaliser in question is quadratic in the data block length, similar to conventional equalisation algorithms, but almost independent of the channel memory length, as opposed to the exponential dependence of the complexity of conventional equalisers on the channel memory length. In [9] it was shown that the binary phase-shift keying (BPSK) version is able to equalise signals in systems with a data block length of 1000 and CIR length of 200. It was also shown in [10] that the quaternary quadrature amplitude modulation (4-QAM) version was able to equalise signals in a UAC with data block lengths of 1200 and CIR lengths of up to 1000, at only double the complexity of its BPSK counterpart. In [11] the model for an iterative 16-QAM MLSE equaliser was derived and simulated, and it was shown that the equaliser effectively equalised uncoded signals in systems with a block length of 500 and CIR length 400, for 16-QAM modulated signals, while optimally recombining the energy spread by the channel, to achieve matched filter performance in a mobile fading environment - something never achieved before in uncoded systems. The significant computational complexity of this equaliser is due to the high parallelism of its underlying recurrent neural network structure, the Hopfield Neural Network (HNN) [12], allowing it to perform equalisation at a fraction of the complexity of conventional optimal equalisers, making it suitable for application in communication systems that have tens or even hundreds of interfering symbols, due long channel delay spread to symbol period ratios.

This paper is organised as follows. The iterative neural network based MLSE equaliser is discussed briefly in Section 2.¹ Section 3 presents a discussion the CIR characteristics in narrow- and wideband communication systems, followed by a discussion on underwater- and CDMA communication systems, as well as a method for exploiting the low complexity equalisation ability of the proposed equaliser in Section 4. Simulation results are presented in Section 5, where after the paper will be concluded with a discussion in Section 6.

II. THE ITERATIVE MLSE EQUALISER

For a single carrier communication systems transmitting N complex symbols $\mathbf{s} = \{s_1, s_2, \dots, s_N\}^T$, chosen from an alphabet \mathcal{D} , through a multipath channel with impulse response $\mathbf{h} = \{h_0, h_1, \dots, h_{L-1}\}$ of length L , the symbol received on the k th instant is described by [1], [4]

$$r_k = \sum_{j=0}^{L-1} h_j s_{k-j} + n_k, \quad (1)$$

where n_k is the k th zero-mean, σ^2 variance, Gaussian noise sample. To find the most likely transmitted sequence \mathbf{s} , the cost function [1]

$$\mathcal{L} = \sum_{k=1}^N \left| r_k - \sum_{j=0}^{L-1} h_j s_{k-j} \right|^2, \quad (2)$$

needs to be minimised. The MLSE equaliser based on the Viterbi Algorithm (VA) [2] minimises (2) optimally by using a trellis, with computational complexity linear in N and exponential in L [1]. The proposed iterative MLSE equaliser also minimises the cost (2), with computational complexity quadratic in N but approximately independent from L , enabling it to perform near-optimal MLSE equalisation in systems with extremely long CIR lengths with very low computational cost.

A. System Model

It was observed that (2) can be written in the form of the HNN [12] energy function

$$\mathcal{L} = -\frac{1}{2} \mathbf{s}^\dagger \mathbf{T} \mathbf{s} - \mathbf{I}^\dagger \mathbf{s}, \quad (3)$$

where \mathbf{I} is an N element column vector, \mathbf{T} is a $N \times N$ correlation matrix, and \dagger is the Hermitian transpose. For complex signal constellations, (3) is rewritten as

$$\mathcal{L} = -\frac{1}{2} [\mathbf{s}_i^T | \mathbf{s}_q^T] \begin{bmatrix} \mathbf{X}_i & \mathbf{X}_q^T \\ \mathbf{X}_q & \mathbf{X}_i \end{bmatrix} \begin{bmatrix} \mathbf{s}_i \\ \mathbf{s}_q \end{bmatrix} - [\mathbf{I}_i^T | \mathbf{I}_q^T] \begin{bmatrix} \mathbf{s}_i \\ \mathbf{s}_q \end{bmatrix}, \quad (4)$$

where \mathbf{s}_i and \mathbf{s}_q denote the respective in-phase and quadrature sequence vector estimates, \mathbf{X}_i and \mathbf{X}_q are $N \times N$ matrices, and \mathbf{I}_i and \mathbf{I}_q are vectors with N elements. \mathbf{X}_i and \mathbf{X}_q are functions of the estimated CIR coefficients, while \mathbf{I}_i and \mathbf{I}_q are functions of the observations $\mathbf{r} = \{r_1, r_2, \dots, r_{N+L-1}\}^T$ and the estimated CIR coefficients. The exact expressions for \mathbf{X}_i , \mathbf{X}_q , \mathbf{I}_i , and \mathbf{I}_q can be found in [11].

By iterating the system equations of (4), given by

$$\begin{aligned} \mathbf{u}^{(n+1)} &= \mathbf{T} \mathbf{s}^{(n)} + \mathbf{I} \\ \mathbf{s}^{(n+1)} &= g(\beta^{(n+1)} \mathbf{u}^{(n+1)}), \end{aligned} \quad (5)$$

where n indicates the iteration number, $g(\beta u_k)$ is a bipolar- or multilevel decision function (see [11]), and $\beta = 1$ is a scaling factor used for optimisation, the system converges to stable state, producing the MLSE estimates in \mathbf{s} . By applying annealing techniques as in [11], the system will escape less optimal minima in order to converge to a near-optimal minimum, producing near-optimal MLSE estimates. A full discussion on the functioning of the iterative M-QAM MSLE equaliser can be found in [11].

III. CHANNEL IMPULSE RESPONSE CHARACTERISATION IN NARROW- AND WIDEBAND SYSTEMS

In a wireless communication system, multiple copies of the originally transmitted signal arrives at the receiver at different time instances, resulting in ISI, corrupting the transmitted signal upon reception. The medium traversed by the signal is called the channel, and in order for the corrupted received signal to equalise at the receiver, the impulse response of the channel, the CIR, needs to be estimated. Fig. 1 shows a continuous channel impulse response in a narrowband communication system. The delay spread τ_s , the time between the first and the last arrival of a transmitted symbol, is 25 μ s.

¹For a complete discussion on the iterative MLSE equaliser, please consult [9], [10], and [11].

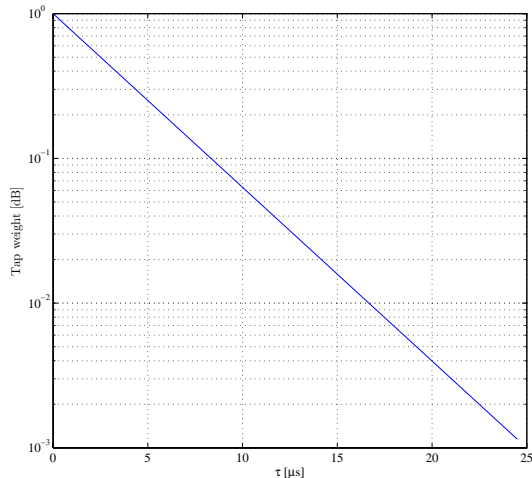


Fig. 1. Continuous channel impulse response.

1) *Narrowband systems:* Assume that the symbol period T_s in a narrowband communication system is $5\mu s$. The CIR length, or the number of CIR coefficients, can be calculated by

$$L = \frac{\tau_s}{T_s}, \quad (6)$$

rounding up to the highest integer, yielding $L = 5$ for this scenario.

Upon demodulation and matched-filtering of the ISI-corrupted received signal, the channel impulse response is estimated by the channel estimator. The channel estimator produces the CIR with length L to be used together with the received sequence of symbols in the equaliser, in order to estimate the sequence of transmitted source symbols with maximum confidence. Fig. 2 shows the CIR produced by the channel estimator for the above scenario. Note that the CIR coefficients are samples of the channel delay spread, taken in the center of the symbol period.

A. Wideband systems

In a wideband communication system, L in (6) increases due to a decrease in the symbol period, given that τ_s remains constant. Assuming that the symbol period for a wideband communication system is $T_s = 0.5\mu s$, given the same delay spread as before, the CIR length is determined by (6) to be $L = 50$. Fig. 3 shows the CIR produced by the channel estimator, where there are now ten times as many CIR coefficients as in the narrowband system.

IV. EQUALISATION IN SYSTEMS WITH LONG CHANNELS

From the discussion in the previous section it should be clear the CIR length L is inversely proportional to the symbol period T_s . Therefore, given a constant channel delay spread τ_s , shorter symbol intervals imply larger CIR lengths, which is more than often the case in CDMA- and underwater communication systems.

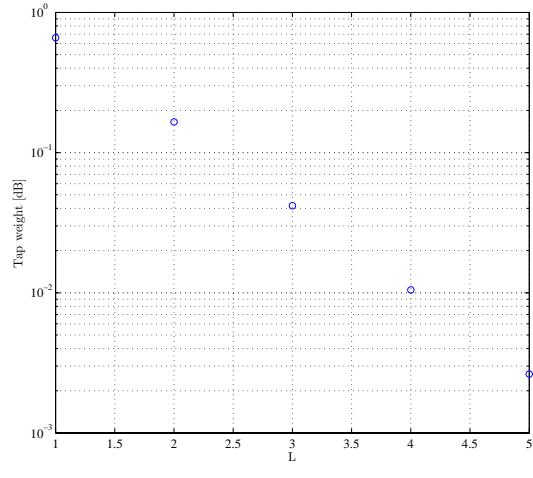


Fig. 2. Channel impulse response for a narrowband system.

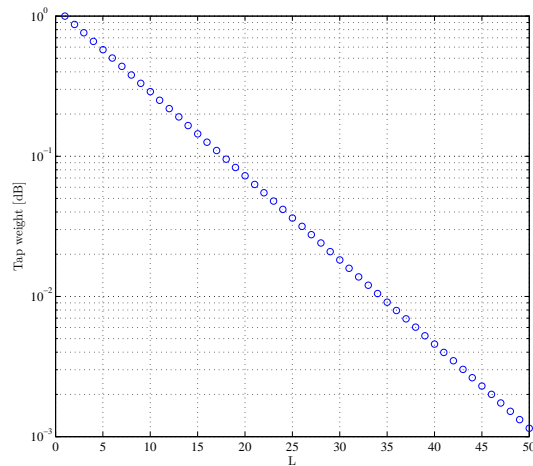


Fig. 3. Channel impulse response for a wideband system.

A. Underwater Acoustic Communication Systems

In an underwater communication systems information is transmitted through the UAC, where it corrupted due to severe ISI and potentially very high Doppler frequencies [13]. Due to the nature of the UAC, and the low speed at which the transmitted signal propagates, there may be hundreds of interfering symbols at the receiver [6], implying CIRs with hundreds of coefficients. In a typical underwater communication system the delay spread can range between 1 ms and 100 ms, potentially extending across hundreds of symbols [6]. Even at moderate data rates, this amount of ISI will render the system unusable.

In an underwater communication system operating at a carrier frequency of $f_c = 15\text{ kHz}$, with a symbol period $T_s = 50\mu s$, where the delay spread duration is 100 ms, the CIR length will be $L = 1000$. Optimal equalisation in this system is impossible due to the large amount of interfering symbols, but the proposed MLSE equaliser is able to equalise signals in this system near-optimally, at very low computational cost compared to conventional optimal equalisers.

Simulation results are presented in [10] as well as in Section 5 of this paper.

B. CDMA communication systems

In CDMA communication systems the signal bandwidth is expanded beyond what is necessary to allow multiple users to transmit data using identical carrier frequencies. This is made possible by the use of orthogonal spreading codes. Each user is assigned a unique code that is used to change the properties of his source data, in order to render it orthogonal to that of other users. After the individual user's source data are spread, the resulting data streams are summed, after which the composite signal is transmitted. At the receiver, this process is reversed by using the users' respective spreading codes to recover each user's source data.

In order for the source data of multiple users to be multiplexed onto one frequency, each user's source symbols of duration T_s are spread by that user's unique spreading sequence, where the duration of the spreading sequence is equal to the symbol period T_s . However, the spreading sequences are series of pulses, or chips, where each chip has a duration T_c . The ratio between the symbol duration and the chip duration,

$$G = \frac{T_s}{T_c}, \quad (7)$$

is known as the spreading- or processing gain.

Assume that K different users are transmitting data on the same frequency, where each user is assigned a unique spreading sequence $c_i^{(j)}$ of length P , where $i = 1, 2, \dots, P$ indicates the elements in the sequence and $j = 1, 2, \dots, K$ indicates the user number. Assuming that the k th BPSK information symbol of user j is denoted by $b_k^{(j)}$, then the received signal in a Gaussian channel can be expressed as

$$z_k = \sum_{j=1}^K \sum_{i=1}^P b_k^{(j)} c_i^{(j)} + n_k, \quad (8)$$

where n_k is a zero mean σ^2 variance Gaussian noise sample. When the composite CDMA signal is subject to multipath, the received signal is expressed as

$$r_k = \sum_{j=0}^{L-1} h_j z_{k-j} + n_k, \quad (9)$$

and to recover the ISI-corrupted signal, a RAKE demodulator is used to coherently recombine the energy that was spread by the channel. The estimated BPSK source symbol k of user j is determined by the sign of

$$\tilde{b}_k^{(j)} = \sum_{i=1}^P c_i^{(j)} \left(\sum_{l=0}^{L-1} r_{k+l} h_l \right), \quad (10)$$

where, as before, \mathbf{h} is the CIR.

C. Proposed Multiple Transmit Antenna Scheme

In order to exploit the capability of the proposed equaliser to equalise signals in systems with hundreds of multipath elements, multiple antennas can be used to transmit the source data.

Consider a conventional single transmit- and receive antenna GSM mobile environment, where the symbol period $T_s = 3.7 \mu\text{s}$, where the channel delay spread spans 7 symbols, yielding a total channel delay of $\tau_{max} = 25.9 \mu\text{s}$ and a CIR length of $L = 7$. Given this setup, multiple transmit antennas, sufficiently spaced apart, can be used to artificially increase the CIR by delaying the signal transmitted by each antenna by multiples of $L = 7$ symbol periods. Therefore, without increasing the energy of each transmitted symbol, the ability of the proposed equaliser to equalise signals in systems with extremely long memory can be exploited to enhance system performance, due to the spatial diversity provided by multiple transmit antennas.

Fig. 4 shows the proposed multiple transmit antenna setup, where the energy of the data-carrying signal is divided between M transmit antennas, where the transmitted signal is delayed by multiples of the channel delay spread τ_{max} according to the corresponding antenna. Assuming that the transmit antennas are sufficiently spaced apart according to the coherence time Δt , each transmitted signal will fade independently. Each transmitted signal therefore propagates through a unique channel, indicated by $h_i(t)$, $i = 1, 2, 3, \dots, M$. Compared to a systems employing only a single transmit antenna, this setup will artificially increase the CIR by a factor M , yielding an effective CIR length of

$$L_{eff} = M \left(\frac{\tau_{max}}{T_s} \right). \quad (11)$$

At the receiver, the extended CIR is estimated as usual to be used together with the received signal to estimate the transmitted source data with maximum confidence, using the proposed iterative MLSE equaliser.

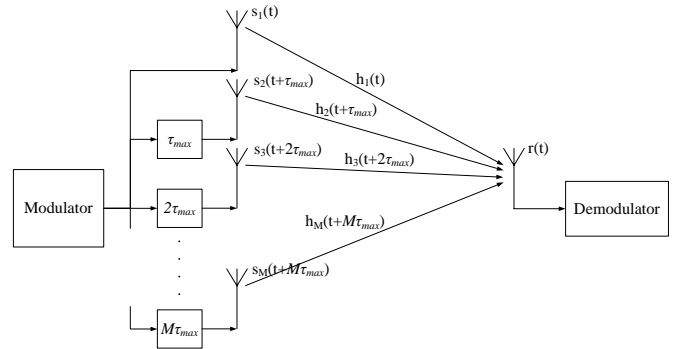


Fig. 4. Proposed multiple transmit antenna setup.

V. SIMULATION

In this section the various applications of the proposed iterative MLSE equaliser, as discussed in this paper, are evaluated via computer simulation. All simulations were performed in a frequency-selective Rayleigh fading environment, where frequency-hopping was employed by generating unique uncorrelated fading vectors for each fading channel, for each transmitted data block, to improve system performance. Also, two different channel models were used in order to accurately characterise the respective simulation environments to produce realistic simulation results. To simulate the fading effect of

each tap, the Rayleigh fading simulator proposed in [14] was used to generate uncorrelated fading vectors. Perfect channel state information (CSI) was assumed, by using the center value of each fading vector - a perfect estimate - to construct the CIR for each data block. $L - 1$ known tail symbols were appended and prepended to the transmitted data block.

A. Channel Models

For the underwater- and CDMA communication systems, an exponential power delay profile is used, and the uniform power delay profile is used to demonstrate the multiple transmit antenna approach. The nominal CIR weights² were chosen as $\mathbf{h} = \left\{ \frac{h_0}{\|\mathbf{h}\|}, \frac{h_1}{\|\mathbf{h}\|}, \dots, \frac{h_{L-1}}{\|\mathbf{h}\|} \right\}$ such that $\mathbf{h}^T \mathbf{h} = 1$, where L is the CIR length, and \mathbf{h} is a column vector of length L .³ After normalisation, the coefficients of \mathbf{h} are used to scale the respective Rayleigh fading vectors used to simulate the multipath effect in order to produce realistic simulation results. The power delay profile models are described in turn below:

1) *Exponential profile*: The exponential decaying power delay profile coefficients are determined by

$$h_k = \exp\left(\frac{-k\tau_{max}}{L\tau_e}\right) \quad (12)$$

where $k = 0, 1, 2, \dots, L-1$, τ_{max} is the channel delay spread duration, and τ_e is the time constant of the profile, determined by

$$\tau_e = \frac{-\tau_{max}}{\ln\left(10^{-\frac{P_{drop}}{10}}\right)}, \quad (13)$$

where P_{drop} is the relative power drop between $t = 0$ and $t = \tau_{max}$, with a value of -30 dB.

2) *Uniform profile*: The linear decaying power delay profile coefficients are determined by

$$h_k = \frac{1}{\sqrt{L}}, \quad (14)$$

where $k = 0, 1, 2, \dots, L-1$.

B. Numerical Results

1) *Underwater Communication System*: The proposed iterative MLSE equaliser was evaluated for a 4-QAM modulation scheme using the exponential channel delay profile. The simulation was carried out for a carrier frequency $f_c = 15$ kHz with symbol interval of $T_s = 50 \mu\text{s}$, for various CIR lengths from $L = 50$ to $L = 1000$, corresponding to delay spreads of 2.5 ms to 50 ms, with a data block length of $N = 1200$ symbols and a Doppler frequency of $F_D = 5$ Hz. Fig. 5 shows the performance of the proposed equaliser for CIR lengths from $L = 50$ to $L = 1000$ in an underwater communication system, compared to the performance of a non-adaptive DFE in each case. The proposed iterative MLSE equaliser clearly outperforms the DFE, as it is able to effectively recombine the ISI-corrupted received symbols, producing near-optimal estimates of the original transmitted symbols. The observed error floors are due to the time variance of the channel, leading to inaccurate channel estimates, and hence degraded system performance.

²The coefficients of \mathbf{h} are determined by the power delay profile.

³For CDMA, the energy is normalised according to the number of users and the spreading gain, and for the multiple antenna scheme, the energy is normalised according to the number of transmit antennas.

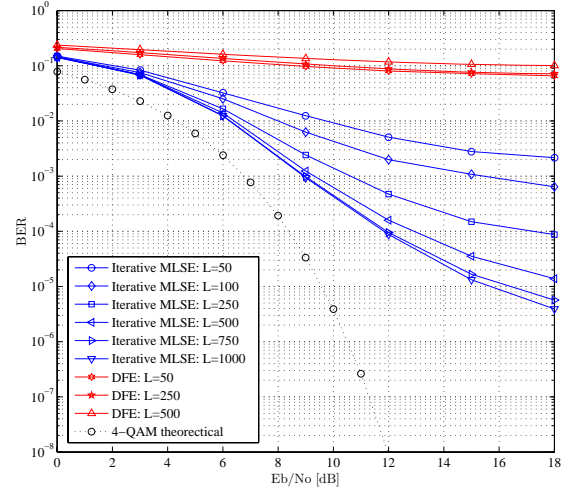


Fig. 5. Performance in an underwater communication system for 4-QAM modulation.

2) *CDMA Communication System*: To evaluate the proposed MLSE equaliser in a CDMA communication system, BPSK modulation was used. Again the exponential channel delay profile was employed to characterise the power delay of the channel. The signal energy was not only divided among the CIR taps, but also among the users. The system was evaluated for BPSK modulation at a carrier frequency of 900 MHz in a system where the symbol period, without spreading, is $T_s = 9.472 \mu\text{s}$, with channel delay spread of $\tau_{max} = 47.36 \mu\text{s}$, corresponding to a CIR length of $L = 5$. By spreading the symbol sequences of each user with spreading sequences, of length $P = 64$, of which each chip in the sequence has a duration $T_c = 0.148 \mu\text{s}$, (7) yields a spreading gain of $G = 64$, corresponding to a CIR of length $L = 320$. The data block length was $N_c = 640$ chips. The spreading sequences were chosen at random from a set of Walsh-Hadamard codes. Fig. 6 shows the performance for the proposed MLSE equaliser and the RAKE demodulator for 1, 2, and 3 users, respectively. It is evident that the proposed MLSE equaliser combines the energy spread by the channel more effectively than the RAKE. This is not surprising, since the RAKE crudely estimates the transmitted chips of the composite signal by recombining the energy of the strongest taps, while the MLSE equaliser uses all the CSI to decorrelate the ISI-corrupted received symbols to produce near-optimal estimates of the transmitted composite chip sequence.

3) *Proposed Multiple Antenna Setup*: By using three multiple transmit antenna setups, the multiple transmit antenna scheme shown in Fig. 4 was evaluated against a system with only one transmit antenna, using the uniform channel delay profile. The system was evaluated for 4-QAM modulation at a carrier frequency of 900 MHz and a symbol period of $T_s = 3.7 \mu\text{s}$, where the channel delay spread $\tau_{max} = 37 \mu\text{s}$, implying a CIR length of $L = 10$ for $M = 1$. The data block length was chosen as $N = 500$ and the mobile speed was set to 3 km/h. To artificially extend the CIR, $M = 2$, $M = 5$ and $M = 10$ transmit antennas were used respectively, to yield CIR lengths of $L = 20$, $L = 50$ and $L = 100$. Fig. 7 shows

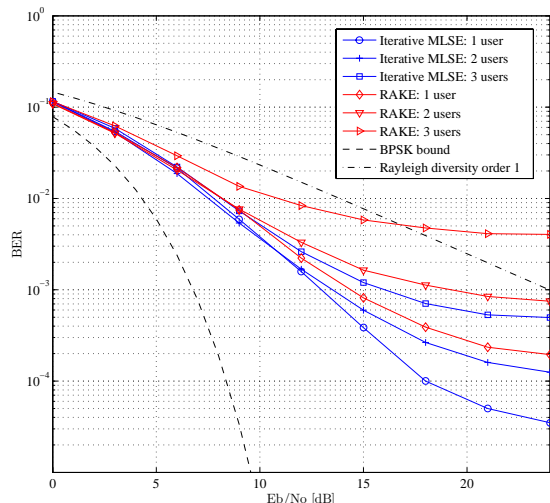


Fig. 6. CDMA performance comparison between the iterative MLSE equaliser and the RAKE for 1, 2, and 3 users.

the performance for systems employing the multiple transmit antenna scheme compared to a normal single transmit antenna system. It is clear that the addition of transmit antennas as in Fig. 4 achieves spatial diversity to artificially extend the CIR, thus exploiting the low complexity equalisation ability of the propose MLSE equaliser. When $L_{eff} = 100$, the iterative MLSE equaliser recombines the energy that was spread across the channel to yield near-optimal 4-QAM performance.

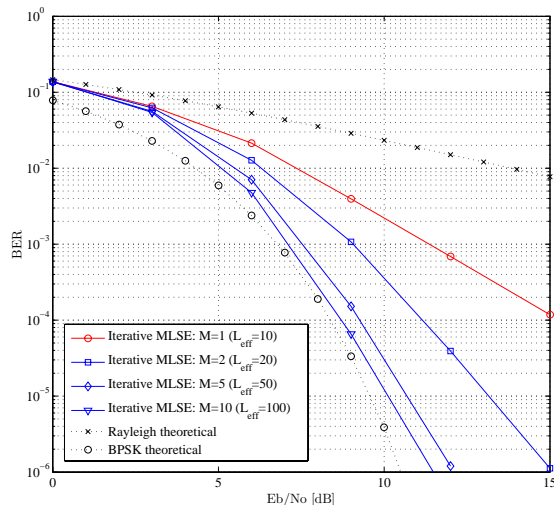


Fig. 7. Multiple transmit antenna performance comparison, for $M = 1$, $M = 2$, $M = 5$, and $M = 10$, using iterative MLSE equaliser.

VI. CONCLUSION

In this paper the low complexity iterative MLSE equaliser, developed by the authors in earlier work, was evaluated for communication systems suffering from ISI caused by hundreds

of interfering symbols. It was shown via computer simulation that the proposed MLSE equaliser effectively equalises signals in underwater- and CDMA communication systems, while an innovative multiple transmit antenna scheme was proposed to utilise the low complexity equalisation ability of the equaliser. It was shown that the equaliser outperforms conventional equalisation methods in both the underwater- and CDMA systems, and the performance gains achieved by the multiple transmit antenna scheme was evident. The low complexity of the proposed equaliser makes it suitable for systems with extremely long memory lengths, where conventional optimal equalisers can neither be applied nor simulated.

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