Review of two management science techniques applied to decision making in the telecommunications industry (Work in progress report)

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Abstract—Modern managers increasingly base their business decisions on the evaluation and interpretation of data. To assist with the decision making process, many mathematical models can be applied. In this study, two specific models viz. Linear Response Surface Analysis (LRSA) and Data Envelopment Analysis (DEA) will be investigated and compared with specific reference to the telecommunications industry. The final objective is to provide a framework that can be used to guide decision makers in selecting the appropriate model under certain circumstances.[1]

Index Terms— Decision making, Data envelopment analysis, Linear response surface analysis

I. INTRODUCTION

MANAGERS are constantly making decisions without knowing what will happen in the future. The success or failure of their businesses depends to a large extent on the quality of their decisions. Often judgment, opinion or past experience is used as a basis for decisions. There are, however, also a great number of mathematical methods available that can aid managers with their decision making responsibilities.

One of the traditional and most popular mathematical models available is regression analysis where the purpose is to understand the relationship between variables and to predict the value of a dependent variable based on the values of the other variables often called regressors [1]. On its own a linear regression model may not always be capable of answering questions concerning the interdependency of variables, influence of specific variables, or optimal values for the dependent variable. To address these questions Bruwer and Hattingh [2] have developed a constrained linear regression technique that can be used to perform linear response surface analysis (LRSA).

Another mathematical approach that is often used as a basis for reviewing and evaluating comparable business or operating units is the so called data envelopment analysis (DEA) technique that was introduced by Charnes, Cooper and Rhodes [3]. This technique is based on variables called inputs and outputs that are used to determine relative efficiency with the objective of revealing differences between inefficient and more efficient operating units. The technique then provides possible targets for the variables of inefficient units. This in turn may then assist managers with decisions necessary to improve those units evaluated as inefficient.

With many different models available for decision making, there is always the danger of not making use of the most appropriate one given certain circumstances. The primary objective of this study is to investigate the applicability of the two techniques (LRSA and DEA) in the telecommunications industry, in order to be able to construct a framework that can be used to guide decision makers when selecting a model under certain given circumstances.

This is a work in progress report. The research has only recently started and this report will present a brief outline of the two techniques to be investigated as well as the planned future work.

II. OVERVIEW OF TECHNIQUES TO BE EVALUATED

A. Data Envelopment Analysis (DEA)

DEA measures the efficiency of each business or operating unit in comparison to a set of units under investigation. This is done by estimating the production function, which relates the inputs consumed to the outputs produced. The model is summarized as follows by Vassiloglou and Giokas [4].

$$\text{Maximise} \quad E_o = \left(\sum_{i=1}^{k} u_i \psi_{io}\right) \Big/ \left(\sum_{j=1}^{m} v_j x_{jo}\right)$$

$$\text{subject to} \quad \left(\sum_{i=1}^{k} u_i \psi_{ir}\right) \Big/ \left(\sum_{j=1}^{m} v_j x_{jr}\right) \leq 1 \qquad r = 1, \ldots, n$$

$$u_i, v_j > \varepsilon \qquad i = 1, \ldots, k \qquad j = 1, \ldots, m$$

where

o = the index of the unit being assessed from the set of r = 1, …, n units

k = the number of outputs at the units

m = the number of inputs at the units

ψ_{ir} = observed output i at unit r

x_{jr} = observed input j at unit r

ε = small positive number

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The above analysis is performed for the different units producing an efficiency rating for each of the *n* units. The required solution is the set of (u_i, v_j) values that maximise the efficiency ratio E_o of the unit being rated without resulting in an input/output ratio exceeding 1 (100% efficiency). Consequently, if a relative efficiency rating of 100% is not attained under this set of weights, it cannot be attained under any other set (for the same sample of units). This fractional programming problem is replaced with a linear programming equivalent through a series of transformations, which are set out in detail in Charnes *et al* [3].

B. Linear Response Surface Analysis

As stated earlier on, a linear regression model may not always provide clear interpretations when used in isolation. To address the shortcomings in certain situations Bruwer and Hattingh [2] have developed a linear response surface analysis technique which, briefly, entails the following.

Firstly, the area of experience, that is, the space enveloped by the available data points is of significance here – this means that the convex hull of available data points is considered. Suppose the following data points, representing measurements on regressors, are available

$$V_i = (X_{i1}, X_{i2}, \ldots.., X_{ik}) \text{ for } i = 1,2,\ldots..,n$$

The convex hull could now be represented by the following set

$$C = \{Z \mid Z \in E^k \text{ and } Z = \sum_{i=1}^{n} \lambda_i V_i \text{ with } \lambda_i \geq 0 \text{ and } \sum_{i=1}^{n} \lambda_i = 1\}$$

where $E^k \equiv k$ - dimensional Euclidean space.

If one assumes that an estimated model in a specific instance is available we will denote the estimated function as $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \ldots... + \hat{\beta}_k X_k$ where the $\hat{\beta}_i$ are obtained by some regression method like least squares Suppose now that the influence of a specific variable, for example X_p, on the dependent variable is required. This variable may be a state variable that is not under the control of the decision maker. It is firstly necessary to know the span covered by X_p within the available data – e.g. if the data is derived from a questionnaire with a 7-point scale, the minimum that may be attained is 1 and the maximum is 7. If we let $X_p = q$ where $q \in (1,7)$, it is possible to determine a maximum/minimum value for the dependent variable $\hat{Y}$ as well as values for the remaining decision variables. The following linear program (parametric in *q*) is now considered.

Maximize $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \ldots.. + \hat{\beta}_k X_k$

subject to

$$\sum_{i=1}^{n} \lambda_i X_{ij} = X_j \text{ for } j = 1,2,\ldots..k$$

$$\sum_{i=1}^{n} \lambda_i = 1, \ \lambda_i \geq 0 \text{ for } i = 1,2,\ldots..n$$

$$X_p = q$$

The solution of this linear programming model then yields the maximum of $\hat{Y}$ as well as the levels of the decision variables $X_1, X_2, \ldots.., X_k$ where this optimum is reached. By solving the problem for various values of q in the interval (1, 7) – for example – the maximum values of $\hat{Y}$ are obtained as well as the sets of optimal levels of the remaining decision variables. These values are then plotted in the form of a graph. A similar procedure is used to obtain minimum values of $\hat{Y}$.

III. Planned Research

This section briefly describes the planned research to be done.

- Perform a comprehensive study of the two management science techniques, LRSA and DEA, to serve as a basis for recommendations.
- Using the two techniques, identify, analyze and compare cases in the telecommunications industry to serve as empirical evidence for conclusions and recommendations.
- Develop a framework for appropriately applying the two techniques in the telecommunications industry.

References

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