Performance Analysis of RSS-Based Geometric Positioning Methods in GSM Networks

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Abstract- Location-based services are fast becoming popular in different areas of the world due to the availability of smart phones and investments by mobile operators in satisfying requests from users. The quality of service (QoS) offered in LBS is dependent on the amount mobile users and the mobile network operators are willing to invest. With feature-rich modern smart phone users get better accuracy using the global positioning system (GPS) or assisted GPS (A-GPS) location method, but unfortunately most users do not have the means to invest in such high-end devices. To ensure improved QoS delivery to mobile stations (MS) with no GPS, the accuracy offered by network-based location methods needs to be reviewed and improved. RSS-based positioning is a network based technique that is the least complex and relatively cost effective since the RSS measurements are readily available in GSM systems. This paper presents a review of received signal strength (RSS) based geometric positioning methods with the intent of exploring the way forward in providing good services to MS with no GPS.

Index Terms— Network-based positioning, LBS, developing countries, RSS, MS, QoS

I. Introduction

The received signal strength (RSS) can be used to estimate the distance between a mobile station (MS) and base transceiver station (BTS). An RSS-based positioning technique is the least complex and relatively most cost effective location method since the RSS measurements are readily available in GSM systems [1]. However, rapidly changing wireless propagation conditions will cause problems in the outdoor environment. In spite of this problem, it is possible to improve the accuracy of this method by either using the received signal level in combination with some other methods or mitigate/eliminate the factors that affect its accuracy. [2, 3]

RSS measurements can be obtained from forward control channels transmitted by the cellular base stations and are implemented in most cellular networks as part of the mobile assisted handoff (MAHO) procedure [2]. This makes the RSS measurement readily available in most networks without hardware changes to the network. It is a general positioning method which can be applied to any cellular or wireless network. It does not rely on the assumption of line of sight (LOS) propagation and turns the multi-path phenomenon into surprisingly good use. [3].

A. Advantage of RSS

The inherent technical difficulties of obtaining accurate measurements of time of arrival (TOA), time difference of arrival (TDOA) and angle of arrival (AOA) makes RSS measurement the only method readily available to be used and have an advantage over the other methods[1, 2, 5]. RSS information is easily available from most types of mobile networks [1, 4]. If the propagation model is known, power measurements can be mapped to distance measurement and once the distance from the MS to three or more BTSs is known, the MS's position can be calculated.

When MSs are used underground, in car parks, and urban canyons, the GPS signal may fail due to the blockage of the satellite-transmitted signals. The GPS has been standardized in Global System for Mobile Communications (GSM) networks as the A-GPS method, which requires hardware development in both the MS and the network [5]. RSS can be used for emergency rescue operations, child tracking, elderly person tracking etc. where RSS can be used to limit the search or positioning to a few square kilometers.

B. Disadvantage of RSS

The major disadvantage of the RSS method is the lower accuracy level it has compared to GPS positioning and can only be used where low accuracies are acceptable or there are no other methods [2, 4]. The inaccuracies introduced by measuring the RSS (power levels) can reach about 30db over a distance of just a wavelength. This introduces low accuracies in the positioning of the user. Another important disadvantage is that the accuracy of the RSS-based methods is highly dependent on the network density, the propagation conditions and the geometry of the network [2]. Accuracy of the range of 5-50m has been achieved indoors and in some urban experiments. Due to the high dependence of the RSS-based method on these highly variable factors, it is very important to estimate the accuracy that can be achieved in a given network before usage [2, 5].

C. Criteria for Location Methods

The following criteria can be used to impartially evaluate and compare different positioning methods and assess their usability. The criteria below will be used to evaluate and define the positioning methods mentioned later [6, 7].

i. Coverage area

The coverage area and capacity for the locating service should be large enough to handle user demands. The coverage area should be a parameter that had been estimated before positioning starts to eliminate non-availability of the LBS to certain users.

ii. Reliability

Reliability is the ratio of accurate and reliable positioning attempts out of all attempts made. It is an important criterion because positioning methods should give high reliability especially ones used in emergencies and asset tracking.

iii. Accurate Positioning

The accuracy needed from positioning is dependent on the LBS required. Some services require very high accuracy i.e. emergency positioning, while some require relatively less accuracy i.e. location-based advertisements. Accuracy is defined by how close the actual location of the MS is to the estimated location.

iv. Applicability

The physical limitation is a very important measure of positioning methods which had not been considered succinctly in relation to usage in developing countries. The physical limitations and requirements associated with the successful implementation of positioning methods in developing countries are mainly network dependency and the amount users can afford to spend on such services.

II. RSS-BASED GEOMETRIC POSITIONING METHODS

A. RSS measurement model/Problem Formulation

The RSS is a signal parameter such that power or energy of a signal travelling between a BTS and a MS contains information related to the distance between them. There is a mathematical relationship between the RSS and the path loss model to estimate the distance [8, 9]. It is assumed that an error free case is considered, where a reliable non line of sight (NLOS) detection algorithm had been used to eliminate measurement with large errors [8] and an averaging of the RSS was carried out over a sufficiently long interval to eliminate the effects of shadowing and multipath fading [9].

Considering two dimensional positioning; let the true location of the MS = $[x_m, y_m]^T$ and the coordinates of the i^{th} BTS $=[x_i, y_i]^T$, i = 1,2 ….N. where N is the total number of transmitting BTSs with adequate receive signal level quality at the MS between 29dBm and -114dBm according to GSM specifications [9].

The distance between the MS and i^{th} BTS, denoted by d_i is given by

$$d_i = \sqrt{(x_m - x_i)^2 + (y_m - y_i)^2} \qquad \ldots 1$$

Where i = 1,2 ……N, number of hearable transmitters

The noise-free RSS or received power at the i^{th} BTS, denoted by P_i^r, is expressed in [8, 10]

$$P_i^r = k_i \frac{P_i^t}{d_i^\alpha}, \ i = 1, 2 \ldots N \qquad \ldots 2$$

Where P_i^t is the transmitted power, P_i^r is the received power, α is the propagation constant, k_i denotes other factors that affect the received power including antenna height, antenna gain, measurement noise etc.

The range related measurement can then be modeled, as expressed in [8]

$$\hat{d}_i = k_i \frac{P_i^t}{d_i^\alpha} + n_i \qquad \ldots 3$$

$$= d_i^\alpha + n_i \qquad \ldots 4$$

$$\hat{d}_i = \left[\sqrt{(x_m - x_i)^2 + (y_m - y_i)^2}\right]^{\frac{\alpha}{2}} + n_i, \ i = 1, 2 \ldots N \qquad \ldots 5$$

n_i = measurement noise in d_i by the i^{th} BTS, α is obtained by finding the path loss slope by measurement, ideally, α is equal to 2 in free space i.e. rural areas and could range from 3 - 6 for urban or suburban [8,10]. The effect of multipath propagation (reflection, diffraction, absorption and refraction) cannot be totally mitigated.

B. Review of some geometric RSS-based methods

The RSS method also makes use of a propagation path loss model which relates the RSS measurement for each BTS at the MS and the relative position of the transmitting and receiving elements with distance [11]. RSS measurements from different BTSs measured at the MS at corresponding times can be used to obtain the distance and the speed of the MS of interest; which can be used to calculate the position of the MS [11, 12]. Using geometric techniques the position of the MS can be estimated. The signal received or rather the level of attenuation of the signal is directly proportional to the distance between the BTS from which the signal is transmitted and the MS of interest. Attenuation levels or the propagation path losses from multiple BTSs are modeled through basic propagation models which best suits the environment of interest [5, 8]. For two-dimensional (2D) positioning each distance measurement provides a radius of a circle centered at the BTS. Measurement from two BTSs will provide an intersection of the circle where the MS is most likely to be located. To resolve ambiguities arising from multiple crossing of lines in the solution provided by just two circles, measurement from at least three BTSs will be needed. [8, 9, 12].

i. Center of Gravity

Zhou et al (2003) proposed the algorithm Center of Gravity (CG) for location estimation as the weighted mean of the locations of the serving BTS and the neighboring BTSs [12]. The CG assumes that the relation between RSS and distance between the MS and the BTS is based on an inverse square law, the relationship was as $R \propto d^{-\alpha}$, where α is the propagation constant caused by the environmental factor, R is the received signal strength and d_i is the distance between the MS and the i^{th} BTS.

Considering that there are i BTSs the MS can receive adequate RSS from; the CG approach defined the location estimation formula as [12],

$$x_m = \frac{x_1 R_1^{-\alpha} + x_2 R_2^{-\alpha} + x_3 R_3^{-\alpha} + \cdots + x_i R_i^{-\alpha}}{R_1^{-\alpha} + R_2^{-\alpha} + R_3^{-\alpha} + \cdots + R_i^{-\alpha}}$$

$$y_m = \frac{y_1 R_1^{-\alpha} + y_2 R_2^{-\alpha} + y_3 R_3^{-\alpha} + \cdots + y_i R_i^{-\alpha}}{R_1^{-\alpha} + R_2^{-\alpha} + R_3^{-\alpha} + \cdots + R_i^{-\alpha}} \qquad \ldots 6$$

Where $[x_m, y_m]^T$ is the estimated location of the MS. $[x_1,y_1]^T$, $[x_2, y_2]^T$, ..., $[x_i, y_i]^T$ are the locations of n serving BTS and the neighboring BTSs. R_1, R_2, ..., R_i are the corresponding RSS received by the MS for each transmitting BTS.

ii. Circular Trilateration (CT)

Zhou et al. (2003) also proposed the CT approach which is accomplished by the construction of 3 circles using the RSS, location of the transmitting BTSs and calculating the intersection of the circles to estimate the location of the MS [12]. It assumes that the relationship between distance from MS to BTS and RSS is based on an inverse square law that is $(N + R)^{\alpha}$ α d, where α is the propagation constant, N is the normalization constant, R_i is the received signal strength at MS from i^{th} BTS. By using this relationship, the CT approach constructed 3 circles as follows, [12, 13].

$$(x_m - x_1)^2 + (y_m - y_1)^2 = \left(\frac{k}{R_1^{\alpha}}\right)^2$$
$$(x_m - x_2)^2 + (y_m - y_2)^2 = \left(\frac{k}{R_2^{\alpha}}\right)^2$$
$$(x_m - x_3)^2 + (y_m - y_3)^2 = \left(\frac{k}{R_3^{\alpha}}\right)^2 \quad \ldots 7$$

Where, R_1, R_2 and R_3 are the RSSs from 3 transmitting BTSs with the geographic locations at $[x_1, y_1]^T$, $[x_2, y_2]^T$ and $[x_3, y_3]^T$ respectively, where k is the common scaling factor [12]. The MS is then estimated as the intersection point of these 3 circles.

iii. Trilateration

Assuming that there are three or more BTSs which the MS can communicate with and receive an acceptable level of signal strength. Considering that the unknown true position of the MS be $[x_m,y_m]^T$ and the BTSs with known coordinates $[x_i,y_i]^T$, i = 1,2,..,N , where N is the number of BTSs under consideration .

Given the distance measurement of each BTS to the receiving MS, the BTS is located at the center of the distance radius and the MS can be located somewhere on the circle. In most cases, two circles will cross each other at two points, producing two solutions, a third circle produced by the third base station normally resolves the ambiguity [14].

Suppose there are four BTSs with known locations $[x_i,y_i]^T$, $1 \le i \le 3$, and that the unknown location of the target is denoted by $[x_m,y_m]^T$. The distance measured from the MS to BTS_i is defined as

$$\hat{d}_i = d_i + n_i \quad , i = 1,2,3 \quad \ldots 8$$

Where n_i is the distance measurement error and

$$d_i = \sqrt{(x_m - x_i)^2 + (y_m - y_i)^2} \quad \ldots 9$$

When there are three unknown location parameters $[x_m,y_m]^T$, using the three distance measurements $\hat{d}_1, \hat{d}_2, \hat{d}_3$ for three BTSs and their known locations, the unknown location parameters $\hat{x}_m, \hat{y}_m$ representing the calculated MS position can be determined using trilateration.

For i = 1,2

$$\hat{y}_m = -\left(\frac{x_2 - x_1}{y_2 - y_1}\right)\hat{x}_m + \frac{\hat{d}_1{}^2 - \hat{d}_2{}^2 + x_2^2 + y_2^2 - (x_1^2 + y_1^2)}{2(y_2 - y_1)} \quad \ldots 10$$

For i = 2,3

$$\hat{y}_m = -\left(\frac{x_2 - x_3}{y_2 - y_3}\right)\hat{x}_m + \frac{\hat{d}_3{}^2 - \hat{d}_2{}^2 + x_2^2 + y_2^2 - (x_3^2 + y_3^2)}{2(y_2 - y_1)} \quad \ldots 11$$

Defining some parameters;

$$\xi_{i,1} = x_i^2 + y_i^2 - (x_1^2 + y_1^2) \quad \ldots 13$$

$$x_{i,j} = x_i - x_j \quad \ldots 14$$

$$y_{i,j} = y_i - y_j \quad \ldots 15$$

here s_{12} is given as

$$s_{1,2} = \frac{1}{2}\left[\left(\hat{d}_1{}^2 - \hat{d}_2{}^2\right) + \xi_{2,1}\right] \quad \ldots 16$$

and s_{13} is given as

$$s_{1,3} = \frac{1}{2}\left[\left(\hat{d}_1{}^2 - \hat{d}_3{}^2\right) + \xi_{3,1}\right] \quad \ldots 17$$

Equations 1.2 and 1.3 can be rearranged to get;

$$\hat{y}_m = \frac{x_{3,1}s_{1,2} - x_{2,1}s_{1,3}}{x_{3,1}y_{2,1} - x_{2,1}y_{3,1}} \quad \ldots 18$$

$$\hat{x}_m = \frac{y_{2,1}s_{1,3} - y_{3,1}s_{1,2}}{x_{3,1}y_{2,1} - x_{2,1}y_{3,1}} \quad \ldots 19$$

iv. Linearization based Least Square method

When the number of independent equations is greater than the number of unknown parameters i.e when there are measurements from more than three BTSs, the LS method can be applied to optimize the redundant measurements to obtain improved location estimates [14].

Consider a network in which there are N BTSs whose coordinates are known $[x_i,y_i]^T$, i = 1,2…N and the coordinates of the MS is denoted by $[x_m,y_m]^T$. When a distance measurement is made to the BTS, there are N measurement equations:

$$\hat{d}_i = d_i + n_i \quad , i = 1,2, \ldots N \quad \ldots 20$$

Where n_i is the distance measurement error and

$$d_i = \sqrt{(x_m - x_i)^2 + (y_m - y_i)^2} \quad \ldots 21$$

A new equation was introduced to linearise the distance measurement equations. Squaring both sides of the equation produces

$$\left(\hat{d}_i - n_i\right)^2 = \sqrt{(x_m - x_i)^2 + (y_m - y_i)^2}$$
$$(x_m{}^2 + y_m{}^2) - (2x_m x_i + 2y_m y_i) = \left(\hat{d}_i - n_i\right)^2 - (x_i^2 + y_i^2) \quad \ldots 22$$

By defining parameters P and P_i as;

$$P = \sqrt{x_m{}^2 + y_m{}^2} \quad \ldots 23$$

$$P_i = \sqrt{x_i{}^2 + y_i{}^2} \quad \ldots 24$$

Equation 23 becomes

$$P^2 - \left(2x_m x_i + 2y_m y_i\right) = \hat{d}_i{}^2 - P_i^2 + n_i{}^2 - 2\hat{d}_i n_i \quad \ldots 25$$

Equation 25 can be written in compact form as

$$\boldsymbol{h} = \boldsymbol{G\theta} + \boldsymbol{n} \quad \ldots 26$$

Where,

$$\boldsymbol{\theta} = [x_m \ \ y_m \ \ P^2]^T \quad \ldots 27$$

$$\boldsymbol{h} = \begin{bmatrix} \hat{d}_1{}^2 - P_1^2 \\ \hat{d}_2{}^2 - P_2^2 \\ \vdots \\ \hat{d}_N{}^2 - P_N^2 \end{bmatrix} \qquad \ldots 28$$

$$\boldsymbol{G} = \begin{bmatrix} -2x_1 & -2y_1 & 1 \\ -2x_2 & -2y_2 & 1 \\ \vdots & \vdots & \vdots \\ -2x_N & -2y_N & 1 \end{bmatrix} \qquad \ldots 29$$

$$\boldsymbol{n} = \; n_1{}^2 - 2\hat{d}_1 n_1 \quad n_2{}^2 - 2\hat{d}_2 n_2 \quad \ldots \quad n_N{}^2 - 2\hat{d}_N n_N \qquad \ldots 30$$

The lease squares solution of equation 25 is

$$\hat{\boldsymbol{\theta}} = \mathbf{G^{-1}h} \qquad \ldots 31$$

C. Evaluation against Criteria

Root Mean Square Error is a widely used to evaluate the accuracy of the location estimates [6, 14].

$$RMSE = \sqrt{\frac{1}{3N}\sum_{m=1}^{N}(\hat{x}_m - x_m)^2 + (\hat{y}_m - y_m)^2} \qquad \ldots 32$$

Where N is the number of location measurements considered, $\hat{x}_m, \hat{y}_m$ are the estimated coordinates, x_m, y_m are the true coordinates of the mobile station of interest and $1 \leq m \leq N$.. Only the successful attempts were considered for the calculation because a few abnormal points could topple the calculations. The RMSE calculation for Centre of Gravity method is the least with approximately 511, followed by Centre of Trilateration method with 1437, the Trilateration method with 3394 and the Linearization based Least Square method error estimate was 271305. Figure 1 shows a graph of the different estimates of the positioning methods and MS's true location for a MS of interest.

Reliability of the positioning method is a ratio of the successful locations to all the attempted location measurements. The trilateration method and the Centre of Trilateration methods were about 46% reliable. They could not provide a unique estimate because the coordinates of the BTSs considered for positioning estimation most likely lie in a straight line. The problem of signal fading in the wireless environment also contributes to their unreliability. The methods that considered more than three BTSs i.e. Linearization based Least Square method and Centre of Gravity were able to produce unique estimations.

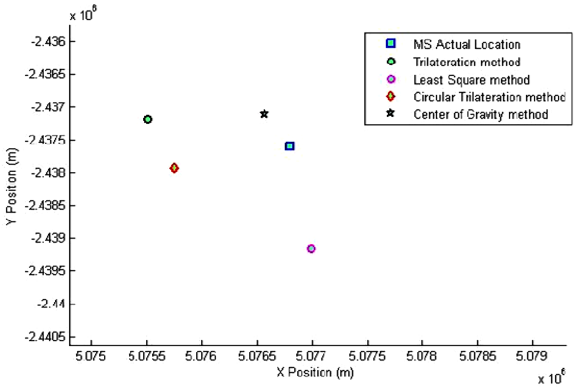

Figure 1 MS location estimates by the positioning methods and its true location for a MS of interest.

The coverage area of the positioning methods is dependent on the coverage areas of BTSs. Once there is a good RSS value from at least three BTSs at the MS, the MS can be located. This makes the RSS-based location method easily assessable to all and cheaper to provide to many users. Table 1 shows comparison between the discussed positioning methods in light of some mentioned criteria.

Table 1: Comparison of discussed positioning methods based on reliability, accuracy and applicability.

Positioning Methods	Reliability	Accuracy	Applicability
Centre of Gravity	High	High	High
Trilateration	Low	Medium	Medium
Linearization with LS	High	Low	Low
Centre of Trilateration	Low	Medium	Medium

All of these methods are applicable in areas of low income earners and where there are no alternative methods. Considering the other mentioned criteria, the most applicable would be the Center of Gravity which is reasonable reliable and accurate. Although it does not meet the US's Federal Communication Commission (FCC) requirement, it can be applied to methods that do not require high accuracy such as climbing emergencies, tracking and network optimization.

D. Conclusion and Future Direction

Although these positioning methods are cheap and relatively easy to implement, their accuracy does not meet the FCC standards and therefore are relatively limited in their usage. Therefore it is important to explore methods to enhance the accuracy and reliability of these methods. Directions for future research in this area can be summarized as follows:

i. *Network topology* - The network topology definitely has an effect on the output of the estimation because different positioning methods and algorithms work in various ways with different topology structures especially in network-based positioning [9]. The network topology as was previously shown with the methods discussed with low reliability plays a major part in the criteria for choosing a positioning method. The error models, multipath propagation models and the NLOS propagation models are highly dependent on the topography and geometry of the BTS and the wireless environment [15].

ii. *Geographical Information System (GIS)* - The wireless environment and base station geometry at different locations in a particular area are not the same. In an urban area there are some areas that appear as rural or suburban. Therefore a hard and fast rule or an urban propagation model cannot be used for every part of an urban area hence the GIS will enhance environment-aware positioning. [15, 16]

iii. *Propagation path loss model*- The propagation path loss models found in literature can be adapted to the environment of interest to improve the accuracy of RSS-based network. Issues that need to be considered include manpower and updating the data frequently. The accuracy of the location estimate depends on the accuracy of the propagation path loss model which in turn depends on the accuracy of data used in tuning it.

References

1. X. Li, "RSS-Based Location Estimation with Unknown Pathloss Model," IEEE Transactions on Wireless Communications, vol. 5, no. 12, pp. 3626-3633, 2006
2. J. Weiss, "On the Accuracy of a Cellular Location System Based on RSS Measurements," IEEE Transactions on Vehicular Technology, vol. 52, no. 6, Pg. 1508-1518
3. B. D. S. Lakmali and D. Dias, "Database Correlation for GSM Location in Outdoor & Indoor Environments," Proceedings of ICIAFS08, 2008
4. O. Bayrak, C. Temizyurek, M. Barut, O. Turkyilmaz and G. Gür, "A Novel Mobile Positioning Algorithm Based on Environment Estimation," 4th Workshop on Positioning, Navigation and Communication (WPNC'07), Hannover, Germany, 2007.
5. H. Laitenen, S. Juurakko, T. Lahti, R. Korhonen and J. Lahteenmaki, " Experimental Evaluation of Location Methods Based on Signal-Strength Measurements," IEEE Transaction on Vehicular Technology, vol. 56, no.1, pp. 287-296, 2007
6. I. K Adusei, K. Kyamakya, K. Jobmann, "Mobile Positioning Technologies in Cellular Networks: An Evaluation of their Performance Metrics," Proceedings of MILCOM 2002, Vol.2, pp... 1239-1244, 2002.
7. M. Silventoinen and T. Rantalainen, (1995) "Mobile Station Locating in GSM," IEEE Wireless Communications System Symposium, Smithtown, pp.. 53-58, 1995.
8. K. W. Cheung H. C. So, W. K. Ma, Y. T. Chan, "Received signal strength based mobile positioning via constrained weighted least squares," Acoustics, Speech, and Signal Processing, 2003. (ICASSP '03). pp, 137-140, 2003.
9. S. Gezici, "A Survey on Wireless Position Estimation," Wireless Personal Communication vol. 44, pp. 263-282, 2008.
10. H. L. Song, "Automatic Vehicle Location in Cellular communications Systems," IEEE Transaction on Vehicular Technology, vo1.43, pp. 902-908, 1994.
11. P. Brida, P. Cepel, J. Duha, "The Accuracy of RSS Based Positioning in GSM Networks," International Conference on Microwaves, Radar & Wireless Communications (MIKON 2006), 2006.
12. J. Zhou, K. M. Chu and J. K. Ng, "Providing Location Services within a Radio Cellular Network using Ellipse Propagation Model," Proceedings of the 19th International Conference on Advanced Information Networking and Applications (AINA'05), 2005
13. J. Zhou, K. M. Chu and J. K. Ng, "A Probabilistic Approach to Mobile Location Estimation within Cellular Networks," Proceedings of the 15th IEEE International Conference on Embedded and Real-Time Computing Systems and Applications, 2009.
14. K. Yu, I. Sharp and Y. J. Guo, "Ground Based Wireless Positioning," IEEE Press Series on Digital and Mobile Communication, John Wiley and Sons Ltd, 2009.
15. F.M. Dahunsi and B. Dwolatzky, "Conceptual Framework that Supports Environment-Aware Positioning and Improved QoS for Location Based Services," Proceedings of IEEE Africa Conference (AFRICON 2009), Nairobi, Kenya
16. R. Filjar, L. Busic, S. Desic, D. Huljenic, "LBS Position Estimation by Adaptive Selection of Positioning sensors Based on Requested QoS," Proceedings of the 8th International Conference on Next Generation Teletraffic and Wired/Wireless Advanced Networking:, Russia, pp. 101-109, 2008.

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