# Characterisation of the Ionospheric Electron Density using Advanced Bayesian Estimation Techniques

Etienne R. Ackermann and Pieter J. de Villiers
Department of Electrical, Electronic and Computer Engineering
University of Pretoria, Pretoria 0002, South Africa
email: etienne.ackermann@ieee.org, pieter.devilliers@up.ac.za
Pierre J. Cilliers Hermanus Magnetic Observatory
P.O. Box 32, Hermanus 7200, South Africa
email: pjcilliers@hmo.ac.za

***Abstract*—The accurate determination of the electron density profile for sections of the ionosphere is of critical importance in a number of scientific and engineering applications, including high frequency (HF) direction finding and navigation. The ionospheric total electron content (TEC) can be derived from dual frequency global positioning satellite (GPS) signals, which may then be used to estimate the electron density profiles through tomographic inversion. This inverse formulation is ill-posed; and leads to an extremely ill-conditioned operator matrix after discretization. This operator matrix is also expected to be highly rank deficient. We will attempt to uniquely reconstruct the electron density distribution by applying nonlinear numerical Bayesian estimation methods such as Gaussian Processes (GP) to the highly multimodal problem.**

## I. INTRODUCTION

The ionosphere is the uppermost part of the Earth's atmosphere, stretching from a height of about 80 km to more than 1000 km. At such heights the atmosphere is so thin that free electrons can exist for short periods of time before they are recombined with nearby positive ions. The level of ionization depends primarily on the received solar radiation, and it exhibits strong diurnal and seasonal variations. The number of these free electrons is sufficient to affect radio wave propagation, which influences the reliability and operation of systems which make use of the earth-ionosphere waveguide [1].

The ionospheric characterisation is also of critical importance in high frequency (HF) direction finding and satellite navigation, since the ionosphere is one of the main error sources in global positioning satellite (GPS) navigation. An accurate estimate of the electron density distribution allows for the correction of an unknown phase delay which is introduced when the GPS $L$-band signals pass through the ionosphere [2].

In recent years dual frequency GPS receivers have been used extensively to monitor the total electron content (TEC) at a large number of locations worldwide (see for example [3]), and a lot of work has been done to use these large amounts of TEC measurements to perform computerised ionospheric tomography (CIT) for the derivation of the electron density distribution spanning wide geographic areas.

However, little progress has been made to facilitate electron density estimation from *single GPS receivers*, since this constitutes a much more difficult problem. Estimation from single receivers is of practical importance mainly in the context of near-real-time characterization of the local ionosphere for HF direction finding, but also for other applications where precision navigation is required such as aircraft takeoff and landing.

## II. PROBLEM STATEMENT

Recently the use of single, dual frequency GPS receivers for CIT was investigated in [4], in which short time series of single satellite, single receiver observations were used successfully in a constrained optimisation formulation to reconstruct the electron density distribution. However, the approach is sensitive to initial estimates and convergence to the global minimum is not guaranteed. This prohibits its usefulness for safety-critical navigation applications.

In this study we will investigate alternative ways to reconstruct the electron density distributions from measurements made by single, dual frequency GPS receivers by the application of advanced numerical Bayesian estimation techniques. We anticipate that these methods will lead to superior reconstruction results, not only ensuring the global optimal, or maximum *a posteriori* density estimates, but also providing our *confidence* in the reconstruction, which will be of immense value to safety-critical systems.

## III. MATHEMATICAL FORMULATION

The ionospheric TEC is defined as the number of free electrons contained in a column of unit cross-sectional area. Mathematically this can be expressed as

$$\text{TEC} = \int\limits_{p_i(s)} N_e(s)\, ds, \tag{1}$$

where $N_e(s)$ is the unknown electron density along the $i$th satellite-receiver ray path $p_i(s)$.

Instead of using the integral equation given in Eq. (1), we will approximate the TEC along some path $p_i$ as a finite sum of integrals along shorter segments of $p_i$. In other words, we will subdivide the path into a set of $N$ layers $\{x_j\}_{j=1}^{N}$, where $x_j$ is the electron density in the $j$th layer.

We can approximate the electron density distribution along a ray path in terms of a number of basis functions $\phi$ from the *Riesz* set through

$$N_e(s) = \sum_{j=1}^{N} x_j \phi_j(\lambda, \psi, z, t) \quad \forall \phi \in \Phi, \tag{2}$$

where $\Phi$ is the truncated *Riesz* basis [5].

In general each basis function can be expressed as a product of functions which represent the spatial and temporal variations of the electron density in the ionosphere [4]:

$$\phi_j(\lambda, \psi, z, t) = H_j(\lambda, \psi) \cdot V_j(z) \cdot T_j(t), \tag{3}$$

where $H_j(\lambda, \psi)$ represents the horizontal (longitude - latitude) variation, $V_j(z)$ represents the variation with altitude, $z$, and $T_j(t)$ represents the time variation.

In this study, the electron density distribution is assumed to be spherically symmetric over the volume spanned by the ray paths for the duration of the observations, so that $H_j(\lambda, \psi) = 1$. We also assume that the electron density distribution changes very slowly relative to the observation period, so that we may set $T_j(t) = 1$.

This reduces Eq. (3) to $\phi_j(\lambda, \psi, z, t) = V_j(z)$, i.e., we will only consider the *vertical electron density* distribution.

We can write the discretized form of Eq. (1) as a linear system of equations,

$$\mathbf{A}\mathbf{x} = \mathbf{b}, \tag{4}$$

where $\mathbf{A} \in \mathbb{R}^{m \times n}$ is the operator matrix with $A_{ij}$ the length of the $i$th ray through the $j$th concentric ionospheric layer. The unknown layer densities is given by $\mathbf{x} \in \mathbb{R}^{n \times 1}$, and $\mathbf{b} \in \mathbb{R}^{m \times 1}$ is the vector of $m$ slant TEC measurements.

The general tomographic inversion procedure then attempts to find $\mathbf{x}^*$ so that

$$f(\mathbf{x}) = \|\mathbf{A}\mathbf{x} - \mathbf{b}\|^2 \tag{5}$$

is minimised, subject to some form of constraint or regularization.

In the Bayesian approach, we will instead consider the *distribution of solutions*, in other words, we will attempt to characterise and solve

$$p(\mathbf{x}|\mathbf{b}) = \frac{p(\mathbf{b}|\mathbf{x}) \cdot p(\mathbf{x})}{p(\mathbf{b})}, \tag{6}$$

from which optimal inferences can be made.

## IV. CONCLUSION

The need for rapid estimation of the electron density distribution in the ionosphere for HF direction finding and other applications is currently limited by the need to combine signals from many dual frequency GPS receivers over an extended period of time. This project will attempt to improve on the speed and accuracy of HF radio direction finding by estimating the electron density distribution in the ionosphere from short time series of GPS navigation data from a single dual frequency GPS receiver. It is expected that Bayesian estimation will provide the best framework on which to do this research, since it allows for optimal decisions to be made by taking all of the known information into account.

## REFERENCES

[1] J.M. Goodman and J. Aarons, “Ionospheric effects on Modern Electronic Systems”, *Proceedings of the IEEE*, vol. 78, no. 3, pp. 512-527, March 1990.

[2] C.N Mitchell and P.S.J. Spencer, “A three-dimensional time-dependent algorithm for ionospheric imaging using GPS,” *Annals of Geophysics*, Vol. 46, no. 4, pp. 687-696, August 2003.

[3] B.M Howe, K. Runciman and J.A. Secan, “Tomography of the ionosphere: Four-dimensional simulations,” *Radio Science*, Vol. 33, no. 1, pp. 109-128, January 1998.

[4] P.J. Cilliers and B.D.L. Oppermann, “Real-time Determination of the Electron Density Distribution in the Ionosphere over South Africa Using Signals from a Single Dual Frequency GPS Receiver,” Research Report Doc No: 6010-0001-712-A1, February 2005

[5] A.J Hansen, T.F. Walter and P.K. Enge, “Real-time Ionospheric Tomography Using Terrestrial GPS Sensors,” in *Proc. of the 11th International Technical Meeting of the Satellite Division of the Institute of Navigation*, Nashville, TN, pp. 717-727, September 1998.

**Etienne R. Ackermann** completed his B.Eng degree at the University of Pretoria in 2008, and is currently studying towards his M.Eng (Electronic Engineering) degree.

**Pieter J. de Villiers** is a lecturer at the Department of Electrical, Electronic and Computer Engineering at the University of Pretoria. His main research interests include statistical signal processing, digital communications, Bayesian inference, machine learning and predictive control. He recently obtained his PhD at the University of Cambridge, England and holds B. Eng and M. Eng (Cum laude) degrees from the University of Pretoria. He is currently the director of the Centre for Telecommunications Engineering in an Information Society (CeTEIS).

**Pierre J. Cilliers** is research physicist in the space physics group of the Hermanus Magnetic Observatory. His main research interests are in ionospheric tomography, ionospheric scintillation and space weather. He holds a MSc in Electrical Engineering from The George Washington University and a PhD in Electrical Engineering from The Ohio State University, both in the USA. He is the principle investigator of the multi-institution project “Polar Space Weather Studies during IPY/IHY”.