

Statistical Estimation of a Fading Channel's Error Probability in a Digital Wireless Communication Network

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Abstract — *This paper proposes a Monte - Carlo based statistical simulation technique in place of the difficult mathematical analysis to obtain an estimate of the error probability of a log-normal Raleigh faded channel in a digital wireless communication network taking into account the users random positions and the modulation type. The results obtained are consistent with those in the literature.*

Index Terms — **Digital Radio Communication, Bit Error Rate, Radio Communication Channel, Monte-Carlo Method.**

I. INTRODUCTION

Communication networks are designed to provide service to their users with acceptable quality level. For that, network traffic should be analyzed and properly controlled so that the desired Quality of Service requirements of applications and Services are achieved. The diversity of applications characteristics and current network technologies demand specific modeling and design tools. Moreover, different techniques including analytical modeling, simulation, measurement and monitoring are required in order to support design and dimensioning of communication networks and services.

One of the key parameters characterising the quality of digital radio communication channel is the bit error rate (BER) which is influenced by the signal to noise ratio. The radio communication channel between stationary and mobile stations are characterised by impairments such as noise, fast and slow fading of the received signal due to multiple users' random movements', multipath fading, inter-symbol interference (ISI), e.t.c. The received signal in the interval $[0, T]$ for a Gaussian channel can be expressed as [1]:

$$r(t) = \alpha(t)e^{-j\phi(t)}s(t) + n(t). \quad (1)$$

where $\alpha(t)$ is the attenuation of received signal amplitude, $s(t)$ is the transmitted signal, $n(t)$ is the Gaussian noise and $\phi(t)$ is the phase of the received signal. Because of the complexity in accounting for the fluctuation of the received signal phase $\phi(t)$, in [1], it is assumed that the phase is constant within the signal interval $[0, T]$, an assumption which is valid for the digital modulation type: FSK, QPSK and QAM.

In order to construct a mathematical model of a radio channel under fading conditions, different statistical distributions are used for the received signal level. In [2], the multipath fading environment is generally modelled as Raleigh or Rician distributions, while in [3, 4], it is shown that experimental data agrees with the m-Nakagami - distribution. In [5], the authors give a unified theoretical analytical approach to BER estimation in a CDMA system for the Nakagami-m fading channel. However, in the above approach, it is assumed that, the channel attenuation is deterministic and that the error is mainly due to multiple access interference (MAI) plus noise, an assumption which might not always be true in a mobile channel. In [6], the authors proposed a close form expression for the average BER by statistically averaging the conditional BER in the case of different independent paths. However, it is difficult to obtain a simple expression even when the paths have the same distribution but with different parameters. The most difficult case occurs when the PDFs of the different paths come from different families of distributions. The Monte Carlo based methodology for estimating the signal and interference powers while accounting for the different random processes is presented in [7, 8]. In [8] a Monte-Carlo based technique for the statistical estimation of signal/noise ratio for a signal under slow fading was proposed. In this paper, we perform a Monte-Carlo base statistical estimation of BER based on the channel's statistical model in which the slow (lognormal) and fast (Raleigh) fading and random movements of the mobile users are taken into account. By using this method, the difficulties in obtaining the PDF of BER as mentioned above are by passed as there is no need for any complicated analytical expression.

In the next section, expressions for the bit energy to noise ratio as a function of bit error rate, for different types of frequently used modulation techniques and the mathematical model for estimating the signal power are presented. Section III presents the models accounting for the fading and shadowing. In section IV the stages involved and the algorithm for a Monte-Carlo based estimation of error probability are presented. In section V, the simulation results for the estimate of the error probability based on the above proposed methodology is presented. Section VI concludes by making a qualitative comparison of the results and also gives a recommendation.

II. BIT ENERGY TO NOISE RATIO

Bit energy to noise ratio (γ_b) at the receiver according to [1], can be expressed in the form:

$$\gamma_b = \alpha^2 \cdot E_b / N_0. \quad (2)$$

where E_b is the mean bit energy and N_0 is the noise spectral density.

The relationship between the bit error rate and the bit energy to noise ratio for different types of frequently used modulation techniques (table 1) in digital communication systems are given in [1].

TABLE 1
BER EXPRESSION FOR VARIOUS MODULATION TYPES

Modulation type	Expression for BER
BPSK	$Q(\sqrt{2\gamma_b})$
DBPSK	$\frac{1}{2} \exp(-\gamma_b)$
M-QAM	$\leq 4Q\left(\sqrt{\frac{3k}{M-1}} \cdot \gamma_b\right)$
BFSK, For coherent detection	$\frac{1}{2} \exp\left(-\frac{\gamma_b}{2}\right)$

In table 1 above,

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{t^2}{2}\right) dt, \quad x \geq 0.$$

The signals level P_{rx} at the receiver is estimated using the mathematical model of the propagation medium as follows:

$$P_{rx} = P_{tx} + G_{tx} + G_{rx} - \eta_{tx} - \eta_{rx} - L. \quad (3)$$

where P_{rx} is the signal power at the receiver (dBm), P_{tx} is the transmitter output power (dBm), G_{tx}, G_{rx} are the transmitter and receiver antenna gains respectively (dB), η_{tx}, η_{rx} are the transmitter and receiver feeder loss respectively (dB), L is the propagation loss (dB). In order to test the results of this work we use the fourth order propagation law [9, 10]:

$$\begin{cases} L|_{r=0} = 10 \log\left(\frac{\lambda^2}{8\pi H_{rx} H_{tx}}\right)^2 - 10 \\ L|_{r \leq r_0} = L|_{r=r_0} - 20 \log(r/r_0) \\ L|_{r > r_0} = L|_{r=r_0} - 38.4 \log(r/r_0) \end{cases}. \quad (4)$$

where $r_0 = 4H_{rx}H_{tx}/\lambda$, H_{tx}, H_{rx} are the transmitter and receiver antenna heights respectively in meters.

III. ACCOUNTING FOR EFFECTS OF SLOW AND FAST FADING

A random process $\alpha_{dB}(t)$ describing the fading in a mobile radio communication channel can be expressed as a sum of independent random processes [11]:

$$\alpha_{dB}(t) = x_{dB}(t) + y_{dB}(t). \quad (5)$$

where, $x_{dB}(t)$ characterises the lognormal distribution with probability density function:

$$f_{x_{dB}} = \frac{1}{\sqrt{2\pi}\sigma} \cdot \exp\left(-\frac{(x_{dB} - \overline{x_{dB}})^2}{2\sigma_x^2}\right),$$

and defines the slowly varying envelop of the received signal $\alpha(t)$, $y_{dB}(t)$ characterises the Raleigh distribution with probability density function:

$$f_{y_{dB}} = \frac{y_{dB}}{\sigma_{y_{dB}}^2} \cdot \exp\left(-\frac{y_{dB}^2}{2\sigma_{y_{dB}}^2}\right),$$

and defines the fast fading of the received signal $\alpha(t)$.

IV. MONTE-CARLO BER ESTIMATION

The error probability (BER) for a particular modulation type, with the statistics of α accounted for can be estimated by the following average [1]:

$$BER = \int_{-\infty}^{\infty} P(\gamma_b) p(\gamma_b) d\gamma_b. \quad (6)$$

where $P(\gamma_b)$ is the BER without considering the random effects of γ_b and $p(\gamma_b)$ is the probability distribution of γ_b . The process of obtaining the final analytical expression for BER from (4) may be mathematically rather complicated or awkward to handle therefore in this case, we implore the use of simulation based on the Monte-Carlo method which bypasses the problem of the analytical awkwardness or complications encountered in the estimation of BER according to (6). The major steps in the Monte-Carlo simulation are shown in Figure 1.

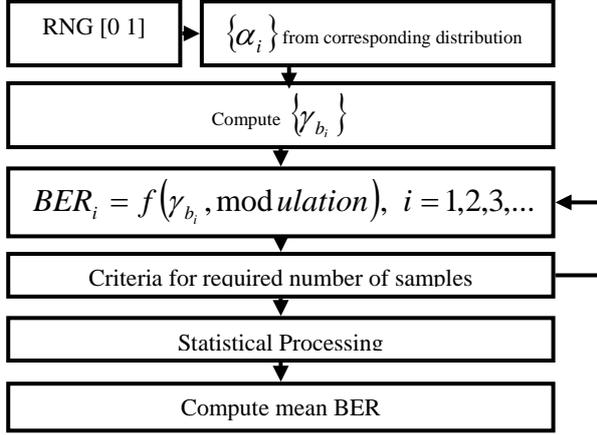


Fig 1. Major steps for BER estimation

A linear congruent generator (RNG) [12] is used for the generation of a series of pseudo random numbers. The calculation of the number of samples was performed in accordance with the Kolmogorov – Smirnov criteria [13].

The exact proposed procedure for a Monte-Carlo based estimation of error probability is given below:

1. Calculate the local mean of the signal \bar{p}_{dBm} (dBm) at the receiver according to (3).
2. Generate the random instantaneous signal level p_{dBm_i} according to the normal distribution with parameters \bar{p}_{dBm} and σ_p (characterising the slowly varying envelop of the received signal)
3. Generate R_i according to the Raleigh distribution with parameter $\sigma_R = \bar{p}/1.253$ (expressed in dB) [10] (characterising the fast fading of the received signal)
4. Calculate the value of α_i^2 as $(10^{p_{dBm_i}/10} \times 10^{R_i/10})^2$ according to (5)
5. Calculate the instantaneous value of the bit energy to noise ratio γ_{b_i} according to (2) for a given E_b/N_0
6. Calculate the bit error rate BER_i corresponding to table 1.
7. Repeat step 2 – 6 a large number (N) times
8. Calculate the mean error probability as

$$\frac{\sum_{i=1}^N BER_i}{N}$$

V. BER ESTIMATION RESULTS

Based on the above presented methodology, the Monte-Carlo simulation results for the dependence of BER on the E_b/N_0 for different modulation types was obtained for the case of uniform distribution of users in a circle of radius Rmax and the results plotted as shown in Figure 2. Typical network parameters used in the simulation are shown in Table 2 below.

TABLE 2
PARAMETERS USED IN THE SIMULATION

Parameter	GSM-900
MS transmit power, dBm	33
BS(MS) antenna height, m	30(1.5)
BS(MS) antenna gain, dB	18(0)
BS(MS) feeder loss, dB	5(0)
Standard deviation, dB	10

VI. CONCLUSION

We have proposed a Monte-Carlo based simulation technique enabling the estimation of the value of the error probability for a digital wireless communication system

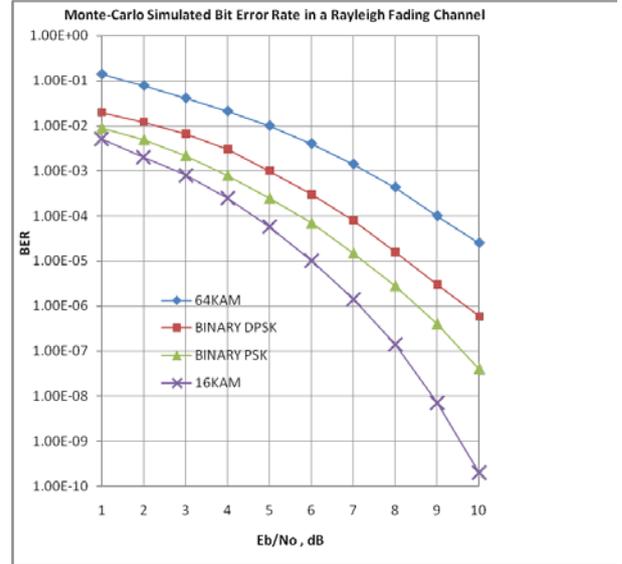


Fig 2. BER vs. E_b/N_0 for the case of uniform distribution of the mobile stations.

with the effects of slow and fast fading and also the users' random position accounted for. Solving this problem analytically can be a very complicated task.

Qualitatively, the estimation results obtained is in agreement with that obtained from experimental data for the multipath urban channel provided in [14]. We could not carry out a quantitative comparison because such comparison requires that the network parameters used in the simulation as in Table 2 above must be common.

These results can be used in the algorithm for the statistical estimation of electromagnetic compatibility of a wireless communication network.

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Nyah Clement TEMANEH was born in Cameroon. He received the Master of Science (MSc.) in Engineering from the Faculty of Radio communication, Radio broadcasting and Television at the Ukrainian State Telecommunications Academy – Odessa in 1995 and a Ph.D. degree in Telecommunications from the Yaroslavl State University – Russia in 2007. Presently he is a lecturer at the University of Namibia. His research interest is on Telecommunication’s network performance simulation models, for network management, planning and optimisation including electromagnetic compatibility analysis.